

Numerical Neural Network Approximations

George A. Anastassiou, Razvan A. Mezei

Numerical Neural Network Approximations

George A. Anastassiou
Department of Mathematical Sciences
University of Memphis
Memphis, TN 38152, U.S.A.

Razvan. A. Mezei
Computer Science Department
Saint Martin's University
Lacey, WA 98503, USA

This monograph is dedicated to our loved ones.

Preface

In this monograph, we present numerical applications of neural networks approximations, as they are presented for the first time in the recent monograph by the first author, titled “Parametrized, Deformed and General Neural networks” [1], Springer, Heidelberg, New York, 2023. That is confirming with numbers the theoretical results of the above mentioned monograph.

Next, we explain very briefly at simplest possible terms why this is of interest in the studies about neural networks approximation. Let h be a general sigmoid function with $h(0) = 0$, and $y = 1$ the horizontal asymptotes. Of course h is strictly increasing over \mathbb{R} . Let the parameter $0 < r < 1$ and $x > 0$. Then clearly $-x < x$ and $-x < -rx < rx < x$, furthermore it holds $h(-x) < h(-rx) < h(rx) < h(x)$. Consequently the sigmoid $y = h(rx)$ has a graph inside the graph of $y = h(x)$, of course with the same asymptotes $y = 1$. Therefore $h(rx)$ has derivatives (gradients) non-zero at more points x than $h(x)$ has different than zero or not as close to zero, thus killing a fewer number of neurons! And of course $h(rx)$ is more distant from $y = 1$, than $h(x)$ is. This is the main concern in choosing the proper activation function, which is a highly desired fact in Neural Networks theory. Also different activation functions allow for different non-linearities which might work better for solving a specific function. So the need to use neural networks with various activation functions is vivid. Thus, performing neural network approximations using different activation functions is not only necessary but fully justified.

Furthermore, the brain non-symmetry has been observed in animals and humans in terms of structure, function and behavior. This lateralization is thought to reflect evolutionary, hereditary, developmental, experiential and pathological factors. Consequently, it is natural to consider for our study deformed neural network activation functions and operators. Thus, this book is the appropriate study covering a great variety of applications and approaching reality as close as possible.

Our numerical applications cover the univariate case extensively in a great number of cases by employing SageMath [3], a free open-source mathematics software that uses a Python-based programming language. We also cover briefly the bivariate case by the use of C# programming language.

In each chapter in short we describe the neural network theory involved there.

The engaged activation function we employ here is the θ -Deformed and λ -parametrized hyperbolic tangent function.

The book's results are expected to find applications in the many areas of Applied Mathematics, Computer Science and Engineering, especially in Artificial Intelligence, Machine Learning and Deep Learning. Other possible applications can be in applied sciences like Statistics, Economics, etc. All in all what is presented here is a valuable tool for a large range of applications. Therefore this monograph is suitable for researchers, graduate students, practitioners and seminars of the above disciplines, also to be in all Science and Engineering libraries.

The preparation of this book took place during 2023-24 at the University of Memphis and Saint Martin's University.

July 1st, 2024.

George A. Anastassiou

Department of Mathematical Sciences

University of Memphis

Memphis, TN 38152, U.S.A.

Razvan. A. Mezei

Computer Science Department

Saint Martin's University

Lacey, WA 98503, USA

Contents

1	Introduction	7
2	Real-valued neural network approximation based on the q-deformed and λ-parametrized Hyperbolic Tangent - introduction	8
3	Real-valued neural network approximation based on the q-deformed and λ-parametrized Hyperbolic Tangent - part 1	9
4	Real-valued neural network approximation based on the q-deformed and λ-parametrized Hyperbolic Tangent - part 2	217
5	Real-valued neural network approximation based on the q-deformed and λ-parametrized Hyperbolic Tangent - part 3	272
6	Real-valued neural network approximation based on the q-deformed and λ-parametrized Hyperbolic Tangent - in the C# programming language	294
7	Bivariate case approximation	334
7.1	Introduction	334
7.2	Using C#	336
7.3	Using SageMath	386
8	Conclusion	390
	References	391

1 Introduction

This work is meant to be a companion to the published monograph of G.A. Anastassiou [Parametrized, Deformed and General Neural Networks, Springer, Heidelberg, New York, 2023], where the theoretical foundations for these results have already been established. In particular, in there, it was proven that the various operators presented below converge quantitatively to the identity operator.

In this current work would like to give our readers a sense of the error of approximation for various operators and particular values of n , parameter λ , deformation coefficient q , activation functions, density functions, neural network operators, and intervals applied for various functions. We estimate the error of approximation of the various neural network operators, and also compare the results to various monomial polynomial functions.

Please note the following:

- the code below was run using SageMath [3] version 9.5 and (towards the end of the book) C# version 10 (.NET 8).
- the estimates in this book are all subject to the error of approximation inherent from using SageMath/C# as a tool and how floating point numbers are represented on computer systems. This error of approximation accumulates as we do more and more iterations.
- some tests are not complete - after running for a long time, a runtime error occurred and some computations stalled.
- the reader is strongly advised to read and understand the code prior to reading the numerical results. The code is written in a simple way so it is understandable even for non computer scientists. The "left side" of our computations represents the error of approximation to the function by neural network operators. This error is compared to anticipated speeds of convergence and expressed by the "difference" values shown in every computational result.

2 Real-valued neural network approximation based on the q -deformed and λ -parametrized Hyperbolic Tangent - introduction

We present in here some of the background and the main result that was proven in the monograph [Parametrized, Deformed and General Neural Networks, Springer, Heidelberg, New York, 2023], in Chapter 18.

The **activation function** [see monograph, formula 18.1] used for this part is defined as follows (note, in the code below, we called this function $\phi(x)$):

$$g_{q,\lambda}(x) := \frac{e^{\lambda x} - qe^{-\lambda x}}{e^{\lambda x} + qe^{-\lambda x}}, \quad \forall x \in \mathbb{R}, \text{ where } q, \lambda > 0. \quad (1)$$

Then [see monograph, formula 18.9], we present the **density function** (which, in the SageMath code below this was named $G(x)$):

$$M_{q,\lambda}(x) := \frac{1}{4}(g_{q,\lambda}(x+1) - g_{q,\lambda}(x-1)) > 0, \quad \forall x \in \mathbb{R}, \text{ where } q, \lambda > 0. \quad (2)$$

Lastly, [see monograph, formula 18.29], we give the real-valued **linear neural network operators**:

$$H_n(f, x) := \frac{\sum_{k=\lceil na \rceil}^{\lfloor nb \rfloor} f\left(\frac{k}{n}\right) M_{q,\lambda}(nx - k)}{\sum_{k=\lceil na \rceil}^{\lfloor nb \rfloor} M_{q,\lambda}(nx - k)}, \quad \text{where } f \in C([a, b]), \quad x \in [a, b], \quad q > 0, \quad q \neq 1. \quad (3)$$

It was shown [see monograph, Theorem 18.9], that:

$$\lim_{n \rightarrow \infty} H_n(f) = f, \quad (4)$$

pointwise and uniformly.

Next, we present our computational results using SageMath. Please note that we removed several of the results generated by the code below.

3 Real-valued neural network approximation based on the q-deformed and λ -parametrized Hyperbolic Tangent - part 1

```
[ ]: RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1]    #deformation parameter lamda over (0,
    ↪ 1] - these are the beta values in the formula
qs = [1/4, 1/2, 1]    #deformation coefficient

funcs = [sin(x), cos(x)]    #choice of functions
a = -pi    #the interval
b = pi    #the interval
x0s= [pi/4, pi/2, 3*pi/4]

#####
for x0 in x0s:
    #####
        for power in powers:    #going over various powers for 1/
            ↪ n^power

            ↪
            ↪#####
                for lamda in lamdas:    #going over each lamda value

                ↪
                ↪#####
                    for q in qs:    #going over each q value

                    ↪
                    ↪#####
                        print()
                        print()

                    ↪
                    ↪#####
                        print("-----")
                        print("x0 = " + str(x0)+", Power = "+
            ↪str(power)+ ", lamda = "+ str(lamda) + ", q = " + str(q))
```

```

    print("-----")

    #the activation function
    phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
    (e^(lamda*x)+q*e^(-lamda*x)) #formula 18.1

    #
    G(x) = 1/4*(phi(x+1) - phi(x-1)) #formula 18.9

    #####
    for i in range(len(funcs)):
        #####
        f(x)=funcs[i]
        show(f(x))
        for n in [10, 20, 50, 100, 200, 500]:
            #def L(n, f, x): #real-valued
            # return sum(f(k/n)*G(n*x-k) for k
            in [ceil(n*a),...,floor(n*b)])/sum(G(n*x-k) for k in
            [ceil(n*a),...,floor(n*b)])
            #leftSide = abs(L(n,f,x0)-f(x0))
            leftSide = abs(sum(f(k/n)*G(n*x0-k)
            for k in [ceil(n*a),...,floor(n*b)])/sum(G(n*x0-k) for k in
            [ceil(n*a),...,floor(n*b)])-f(x0))
            val1 = n
            val2 = leftSide.n()
            val3 = 1/(n^power).n()
            print("n = "+str(val1), "1/
            left side = "+str(val2), "\n
            n^("+str(power)+") = "+str(val3), "\n
            difference = "+str(val3-val2))

```

 $x_0 = 1/4\pi$, Power = 3/10, lamda = 1/4, q = 1/4

$\sin(x)$

```

n = 10 , left side = 1.10269377456684e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.90917856170588e-1
n = 20 , left side = 7.76237576737798e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.29466773863125e-1
n = 50 , left side = 3.60961153613114e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.73153379349680e-1
n = 100 , left side = 1.88410272392819e-2
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.32347615911676e-1
n = 200 , left side = 9.61343382830937e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.94415143508528e-1
n = 500 , left side = 3.89095422715457e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.51100944527679e-1

```

$\cos(x)$

```

n = 10 , left side = 2.51869405006841e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 2.49317828620431e-1
n = 20 , left side = 1.14542456854608e-1
          1/n^(3/10) = 4.07090531536904e-1
          difference = 2.92548074682297e-1
n = 50 , left side = 4.20732317898758e-2

```

```

1/n^(3/10) = 3.09249494710992e-1
difference = 2.67176262921116e-1
n = 100 , left side = 2.03378378667151e-2
1/n^(3/10) = 2.51188643150958e-1
difference = 2.30850805284243e-1
n = 200 , left side = 9.98779497231417e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.94040782364523e-1
n = 500 , left side = 3.95085911348803e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.51041039641346e-1

```

 $x_0 = 1/4\pi$, Power = 3/10, lamda = 1/4, q = 1/2

$\sin(x)$

```

n = 10 , left side = 3.95100600139320e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.61677173613340e-1
n = 20 , left side = 3.46988342396717e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.72391697297233e-1
n = 50 , left side = 1.73741914965689e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.91875303214423e-1
n = 100 , left side = 9.25093740382410e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.41937705747134e-1
n = 200 , left side = 4.76419447265530e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.99264382864182e-1
n = 500 , left side = 1.93866173914381e-3
1/n^(3/10) = 1.54991898754834e-1

```

difference = 1.53053237015690e-1

$\cos(x)$

n = 10 , left side = 1.43310623580410e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.57876610046862e-1
n = 20 , left side = 6.16155530779091e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.45474978458995e-1
n = 50 , left side = 2.17255095716978e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.87523985139294e-1
n = 100 , left side = 1.03403776602996e-2
1/n^(3/10) = 2.51188643150958e-1
difference = 2.40848265490658e-1
n = 200 , left side = 5.03665536910569e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.98991921967731e-1
n = 500 , left side = 1.98226000171708e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53009638753117e-1

x0 = 1/4*pi, Power = 3/10, lamda = 1/4, q = 1

$\sin(x)$

n = 10 , left side = 4.55658979518567e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.55621335675416e-1
n = 20 , left side = 1.17886898348851e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.95301841702019e-1

```

n = 50 , left side = 1.90462319911511e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.07344871511877e-1
n = 100 , left side = 4.76820716817761e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50711822434140e-1
n = 200 , left side = 1.19246801639550e-4
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03909330535197e-1
n = 500 , left side = 1.90813536956602e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54972817401138e-1

```

$\cos(x)$

```

n = 10 , left side = 4.55601836086842e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.55627050018588e-1
n = 20 , left side = 1.17886897702854e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.95301841766619e-1
n = 50 , left side = 1.90462319911577e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.07344871511876e-1
n = 100 , left side = 4.76820716817428e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50711822434141e-1
n = 200 , left side = 1.19246801639439e-4
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03909330535198e-1
n = 500 , left side = 1.90813536956602e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54972817401138e-1

```

$x_0 = 1/4\pi$, Power = 3/10, lamda = 1/2, q = 1/4

$\sin(x)$

n = 10 , left side = 7.66430508778090e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.24544182749463e-1
n = 20 , left side = 4.38693273663759e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.63221204170528e-1
n = 50 , left side = 1.88047297835471e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.90444764927445e-1
n = 100 , left side = 9.60447473250947e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.41584168418449e-1
n = 200 , left side = 4.85201986681627e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.99176557470021e-1
n = 500 , left side = 1.95265780521114e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53039240949623e-1

$\cos(x)$

n = 10 , left side = 1.15283010872480e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.85904222754792e-1
n = 20 , left side = 5.36357377250527e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.53454793811852e-1
n = 50 , left side = 2.03721756047669e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.88877319106225e-1
n = 100 , left side = 9.99650884011438e-3

```

1/n^(3/10) = 2.51188643150958e-1
difference = 2.41192134310844e-1
n = 200 , left side = 4.95003918172598e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.99078538155111e-1
n = 500 , left side = 1.96834138039348e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53023557374440e-1

```

 $x_0 = 1/4\pi$, Power = 3/10, lamda = 1/2, q = 1/2

$\sin(x)$

```

n = 10 , left side = 3.37717704676072e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.67415463159665e-1
n = 20 , left side = 2.07753148869365e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.86315216649968e-1
n = 50 , left side = 9.21511967883226e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.00034375032159e-1
n = 100 , left side = 4.75529633802174e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.46433346812936e-1
n = 200 , left side = 2.41425898359460e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.01614318353242e-1
n = 500 , left side = 9.74447341997586e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54017451412836e-1

```

$\cos(x)$


```

n = 10 , left side = 6.24222476166085e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.38764986010664e-1
n = 20 , left side = 2.80065121964507e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.79084019340454e-1
n = 50 , left side = 1.03752156077472e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.98874279103244e-1
n = 100 , left side = 5.04543146629388e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.46143211684664e-1
n = 200 , left side = 2.48679971416887e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.01541777622668e-1
n = 500 , left side = 9.86054169944683e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54005844584889e-1

```

 $x_0 = 1/4\pi$, Power = 3/10, lamda = 1/2, q = 1

$\sin(x)$

```

n = 10 , left side = 1.26576588370484e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.88529574790224e-1
n = 20 , left side = 3.19289527212985e-3
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.03897636264775e-1
n = 50 , left side = 5.12151444058917e-4
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.08737343266933e-1
n = 100 , left side = 1.28084179052301e-4

```

```

1/n^(3/10) = 2.51188643150958e-1
difference = 2.51060558971906e-1
n = 200 , left side = 3.20238618125579e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03996553475024e-1
n = 500 , left side = 5.12400106722488e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54986774753766e-1

```

$\cos(x)$

```

n = 10 , left side = 1.26576548275704e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.88529578799702e-1
n = 20 , left side = 3.19289294996883e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.03897638586936e-1
n = 50 , left side = 5.12152389308573e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.08737342321683e-1
n = 100 , left side = 1.28084061290612e-4
1/n^(3/10) = 2.51188643150958e-1
difference = 2.51060559089667e-1
n = 200 , left side = 3.20239758674346e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03996553360970e-1
n = 500 , left side = 5.12391080254027e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54986774844031e-1

```

$x_0 = 1/4\pi$, Power = 3/10, lamda = 1, q = 1/4

$\sin(x)$

$n = 10$, left side = $4.29282034779491e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.58259030149323e-1$
 $n = 20$, left side = $2.30286683697076e-2$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $3.84061863167197e-1$
 $n = 50$, left side = $9.56745400404757e-3$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $2.99682040706944e-1$
 $n = 100$, left side = $4.84538571465232e-3$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.46343257436306e-1$
 $n = 200$, left side = $2.43500798502960e-3$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.01593569351807e-1$
 $n = 500$, left side = $9.78230233303456e-4$
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = $1.54013668521530e-1$

$$\cos(x)$$

$n = 10$, left side = $5.44419628451549e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.46745270782117e-1$
 $n = 20$, left side = $2.59171447278922e-2$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $3.81173386809012e-1$
 $n = 50$, left side = $1.00304303028137e-2$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $2.99219064408178e-1$
 $n = 100$, left side = $4.96112082798739e-3$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.46227522322971e-1$
 $n = 200$, left side = $2.46392751821078e-3$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.01564649818626e-1$

```

n = 500 , left side = 9.82856796759912e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54009041958074e-1

```

```

-----
x0 = 1/4*pi, Power = 3/10, lamda = 1,  q = 1/2
-----

```

$\sin(x)$

```

n = 10 , left side = 1.98858419604222e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.81301391666850e-1
n = 20 , left side = 1.11111242904842e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.95979407246420e-1
n = 50 , left side = 4.71676040228819e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.04532734308704e-1
n = 100 , left side = 2.40386308996299e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.48784780060995e-1
n = 200 , left side = 1.21454789680442e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.02814029440033e-1
n = 500 , left side = 4.88452065581013e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54503446689253e-1

```

$\cos(x)$

```

n = 10 , left side = 2.88791443525872e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.72308089274685e-1
n = 20 , left side = 1.33664407456531e-2

```

```

1/n^(3/10) = 4.07090531536904e-1
difference = 3.93724090791251e-1
n = 50 , left side = 5.07733367488394e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.04172161036108e-1
n = 100 , left side = 2.49412468567678e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.48694518465281e-1
n = 200 , left side = 1.23708851068816e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.02791488826149e-1
n = 500 , left side = 4.92063839226731e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54499834915607e-1

```

x0 = 1/4*pi, Power = 3/10, lamda = 1, q = 1

```

sin(x)
n = 10 , left side = 4.09324613380091e-3
1/n^(3/10) = 5.01187233627272e-1
difference = 4.97093987493471e-1
n = 20 , left side = 1.03161560946519e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.06058915927439e-1
n = 50 , left side = 1.58852765311801e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.09090641945680e-1
n = 100 , left side = 4.13962851463223e-5
1/n^(3/10) = 2.51188643150958e-1
difference = 2.51147246865812e-1
n = 200 , left side = 9.67248590255654e-6
1/n^(3/10) = 2.04028577336837e-1

```

```

difference = 2.04018904850934e-1
n = 500 , left side = 2.07035881660822e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54989828396017e-1

```

$\cos(x)$

```

n = 10 , left side = 4.05635916216029e-3
1/n^(3/10) = 5.01187233627272e-1
difference = 4.97130874465112e-1
n = 20 , left side = 1.00937393504252e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.06081157601862e-1
n = 50 , left side = 1.67976724238628e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.09081517986753e-1
n = 100 , left side = 4.02584365390979e-5
1/n^(3/10) = 2.51188643150958e-1
difference = 2.51148384714419e-1
n = 200 , left side = 1.07750350520908e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.04017802301785e-1
n = 500 , left side = 1.19782555663139e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54990700929277e-1

```

$x_0 = 1/4\pi$, Power = 1/2, lamda = 1/4, q = 1/4

$\sin(x)$

```

n = 10 , left side = 1.10269377456684e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 2.05958388560154e-1

```

$n = 20$, left side = $7.76237576737798e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $1.45983040076199e-1$
 $n = 50$, left side = $3.60961153613114e-2$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $1.05325240875998e-1$
 $n = 100$, left side = $1.88410272392819e-2$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $8.11589727607181e-2$
 $n = 200$, left side = $9.61343382830937e-3$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $6.10972442903454e-2$
 $n = 500$, left side = $3.89095422715457e-3$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.08304053228412e-2$

$$\cos(x)$$

$n = 10$, left side = $2.51869405006841e-1$
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = $6.43583610099968e-2$
 $n = 20$, left side = $1.14542456854608e-1$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $1.09064340895371e-1$
 $n = 50$, left side = $4.20732317898758e-2$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $9.93481244474337e-2$
 $n = 100$, left side = $2.03378378667151e-2$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $7.96621621332849e-2$
 $n = 200$, left side = $9.98779497231417e-3$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $6.07228831463406e-2$
 $n = 500$, left side = $3.95085911348803e-3$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.07705004365078e-2$

 $x_0 = 1/4\pi$, Power = 1/2, lamda = 1/4, q = 1/2

$\sin(x)$

```

n = 10 , left side = 3.95100600139320e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.76717706002906e-1
n = 20 , left side = 3.46988342396717e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.88907963510307e-1
n = 50 , left side = 1.73741914965689e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.24047164740741e-1
n = 100 , left side = 9.25093740382410e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.07490625961759e-2
n = 200 , left side = 4.76419447265530e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.59464836459994e-2
n = 500 , left side = 1.93866173914381e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.27826978108520e-2

```

$\cos(x)$

```

n = 10 , left side = 1.43310623580410e-1
          1/n^(1/2) = 3.16227766016838e-1
          difference = 1.72917142436428e-1
n = 20 , left side = 6.16155530779091e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.61991244672070e-1
n = 50 , left side = 2.17255095716978e-2

```



```

1/n^(1/2) = 1.41421356237310e-1
difference = 1.19695846665612e-1
n = 100 , left side = 1.03403776602996e-2
1/n^(1/2) = 1.00000000000000e-1
difference = 8.96596223397004e-2
n = 200 , left side = 5.03665536910569e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.56740227495491e-2
n = 500 , left side = 1.98226000171708e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.27390995482787e-2

```

 $x_0 = 1/4\pi$, Power = 1/2, lamda = 1/4, q = 1

```

sin(x)
n = 10 , left side = 4.55658979518567e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.70661868064981e-1
n = 20 , left side = 1.17886898348851e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.11818107915094e-1
n = 50 , left side = 1.90462319911511e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.39516733038194e-1
n = 100 , left side = 4.76820716817761e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.95231792831822e-2
n = 200 , left side = 1.19246801639550e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.05914313170152e-2
n = 500 , left side = 1.90813536956602e-5
1/n^(1/2) = 4.47213595499958e-2

```

difference = 4.47022781963001e-2

$\cos(x)$

n = 10 , left side = 4.55601836086842e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.70667582408154e-1
n = 20 , left side = 1.17886897702854e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.11818107979694e-1
n = 50 , left side = 1.90462319911577e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.39516733038194e-1
n = 100 , left side = 4.76820716817428e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.95231792831826e-2
n = 200 , left side = 1.19246801639439e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.05914313170153e-2
n = 500 , left side = 1.90813536956602e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47022781963001e-2

x0 = 1/4*pi, Power = 1/2, lamda = 1/2, q = 1/4

$\sin(x)$

n = 10 , left side = 7.66430508778090e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.39584715139029e-1
n = 20 , left side = 4.38693273663759e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.79737470383603e-1

```

n = 50 , left side = 1.88047297835471e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.22616626453762e-1
n = 100 , left side = 9.60447473250947e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.03955252674905e-2
n = 200 , left side = 4.85201986681627e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.58586582518385e-2
n = 500 , left side = 1.95265780521114e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.27687017447846e-2

```

$\cos(x)$

```

n = 10 , left side = 1.15283010872480e-1
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.00944755144358e-1
n = 20 , left side = 5.36357377250527e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.69971060024926e-1
n = 50 , left side = 2.03721756047669e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.21049180632543e-1
n = 100 , left side = 9.99650884011438e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.00034911598856e-2
n = 200 , left side = 4.95003918172598e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.57606389369288e-2
n = 500 , left side = 1.96834138039348e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.27530181696023e-2

```

$x_0 = 1/4\pi$, Power = 1/2, lamda = 1/2, q = 1/2

$\sin(x)$

n = 10 , left side = 3.37717704676072e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.82455995549231e-1
n = 20 , left side = 2.07753148869365e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.02831482863042e-1
n = 50 , left side = 9.21511967883226e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.32206236558477e-1
n = 100 , left side = 4.75529633802174e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.52447036619783e-2
n = 200 , left side = 2.41425898359460e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.82964191350602e-2
n = 500 , left side = 9.74447341997586e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.37469122079982e-2

$\cos(x)$

n = 10 , left side = 6.24222476166085e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.53805518400229e-1
n = 20 , left side = 2.80065121964507e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.95600285553528e-1
n = 50 , left side = 1.03752156077472e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.31046140629562e-1
n = 100 , left side = 5.04543146629388e-3

```

1/n^(1/2) = 1.000000000000000e-1
difference = 9.49545685337061e-2
n = 200 , left side = 2.48679971416887e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.82238784044859e-2
n = 500 , left side = 9.86054169944683e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.37353053800511e-2

```

x0 = 1/4*pi, Power = 1/2, lamda = 1/2, q = 1

```

sin(x)
n = 10 , left side = 1.26576588370484e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 3.03570107179790e-1
n = 20 , left side = 3.19289527212985e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.20413902477849e-1
n = 50 , left side = 5.12151444058917e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.40909204793251e-1
n = 100 , left side = 1.28084179052301e-4
1/n^(1/2) = 1.000000000000000e-1
difference = 9.98719158209477e-2
n = 200 , left side = 3.20238618125579e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06786542568422e-2
n = 500 , left side = 5.12400106722488e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47162355489286e-2

```

```

cos(x)

```

```

n = 10 , left side = 1.26576548275704e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 3.03570111189268e-1
n = 20 , left side = 3.19289294996883e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.20413904800010e-1
n = 50 , left side = 5.12152389308573e-4
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.40909203848001e-1
n = 100 , left side = 1.28084061290612e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.98719159387094e-2
n = 200 , left side = 3.20239758674346e-5
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.06786541427873e-2
n = 500 , left side = 5.12391080254027e-6
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47162356391933e-2

```

 $x_0 = 1/4\pi$, Power = 1/2, lamda = 1, q = 1/4

$\sin(x)$

```

n = 10 , left side = 4.29282034779491e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.73299562538889e-1
n = 20 , left side = 2.30286683697076e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.00578129380271e-1
n = 50 , left side = 9.56745400404757e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.31853902233262e-1
n = 100 , left side = 4.84538571465232e-3

```

```

1/n^(1/2) = 1.000000000000000e-1
difference = 9.51546142853477e-2
n = 200 , left side = 2.43500798502960e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.82756701336251e-2
n = 500 , left side = 9.78230233303456e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.37431293166923e-2

```

$\cos(x)$

```

n = 10 , left side = 5.44419628451549e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.61785803171683e-1
n = 20 , left side = 2.59171447278922e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.97689653022087e-1
n = 50 , left side = 1.00304303028137e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.31390925934496e-1
n = 100 , left side = 4.96112082798739e-3
1/n^(1/2) = 1.000000000000000e-1
difference = 9.50388791720126e-2
n = 200 , left side = 2.46392751821078e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.82467506004440e-2
n = 500 , left side = 9.82856796759912e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.37385027532359e-2

```

 $x_0 = 1/4\pi$, Power = 1/2, lamda = 1, q = 1/2

$\sin(x)$

$n = 10$, left side = 1.98858419604222e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.96341924056416e-1
 $n = 20$, left side = 1.11111242904842e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 2.12495673459495e-1
 $n = 50$, left side = 4.71676040228819e-3
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.36704595835021e-1
 $n = 100$, left side = 2.40386308996299e-3
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.75961369100370e-2
 $n = 200$, left side = 1.21454789680442e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.94961302218503e-2
 $n = 500$, left side = 4.88452065581013e-4
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.42329074844148e-2

$$\cos(x)$$

$n = 10$, left side = 2.88791443525872e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.87348621664251e-1
 $n = 20$, left side = 1.33664407456531e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 2.10240357004326e-1
 $n = 50$, left side = 5.07733367488394e-3
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.36344022562426e-1
 $n = 100$, left side = 2.49412468567678e-3
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.75058753143232e-2
 $n = 200$, left side = 1.23708851068816e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.94735896079666e-2


```

n = 500 , left side = 4.92063839226731e-4
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.42292957107691e-2

```

```

-----
x0 = 1/4*pi, Power = 1/2, lamda = 1,  q = 1
-----

```

$\sin(x)$

```

n = 10 , left side = 4.09324613380091e-3
          1/n^(1/2) = 3.16227766016838e-1
          difference = 3.12134519883037e-1
n = 20 , left side = 1.03161560946519e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.22575182140514e-1
n = 50 , left side = 1.58852765311801e-4
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.41262503471998e-1
n = 100 , left side = 4.13962851463223e-5
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.99586037148537e-2
n = 200 , left side = 9.67248590255654e-6
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.07010056327522e-2
n = 500 , left side = 2.07035881660822e-6
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47192891911792e-2

```

$\cos(x)$

```

n = 10 , left side = 4.05635916216029e-3
          1/n^(1/2) = 3.16227766016838e-1
          difference = 3.12171406854678e-1
n = 20 , left side = 1.00937393504252e-3

```

```

1/n^(1/2) = 2.23606797749979e-1
difference = 2.22597423814936e-1
n = 50 , left side = 1.67976724238628e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41253379513071e-1
n = 100 , left side = 4.02584365390979e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99597415634609e-2
n = 200 , left side = 1.07750350520908e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06999030836027e-2
n = 500 , left side = 1.19782555663139e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47201617244392e-2

```

 $x_0 = 1/4\pi$, Power = 7/10, lamda = 1/4, q = 1/4

```

sin(x)
n = 10 , left side = 1.10269377456684e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 8.92568540402040e-2
n = 20 , left side = 7.76237576737798e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 4.51990449377992e-2
n = 50 , left side = 3.60961153613114e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 2.85765852964243e-2
n = 100 , left side = 1.88410272392819e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 2.09696898160678e-2
n = 200 , left side = 9.61343382830937e-3
1/n^(7/10) = 2.45063709469745e-2

```

```

        difference = 1.48929371186651e-2
n = 500 , left side = 3.89095422715457e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 9.01294601580975e-3

```

$\cos(x)$

```

n = 10 , left side = 2.51869405006841e-1
        1/n^(7/10) = 1.99526231496888e-1
        difference = -5.23431735099532e-2
n = 20 , left side = 1.14542456854608e-1
        1/n^(7/10) = 1.22822802611579e-1
        difference = 8.28034575697120e-3
n = 50 , left side = 4.20732317898758e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 2.25994688678600e-2
n = 100 , left side = 2.03378378667151e-2
        1/n^(7/10) = 3.98107170553497e-2
        difference = 1.94728791886347e-2
n = 200 , left side = 9.98779497231417e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 1.45185759746603e-2
n = 500 , left side = 3.95085911348803e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 8.95304112947630e-3

```

$x_0 = 1/4\pi$, Power = 7/10, lamda = 1/4, q = 1/2

$\sin(x)$

```

n = 10 , left side = 3.95100600139320e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.60016171482956e-1

```

$n = 20$, left side = $3.46988342396717e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $8.81239683719074e-2$
 $n = 50$, left side = $1.73741914965689e-2$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $4.72985091611668e-2$
 $n = 100$, left side = $9.25093740382410e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.05597796515256e-2$
 $n = 200$, left side = $4.76419447265530e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $1.97421764743192e-2$
 $n = 500$, left side = $1.93866173914381e-3$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.09652385038205e-2$

$\cos(x)$

$n = 10$, left side = $1.43310623580410e-1$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $5.62156079164780e-2$
 $n = 20$, left side = $6.16155530779091e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $6.12072495336699e-2$
 $n = 50$, left side = $2.17255095716978e-2$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $4.29471910860379e-2$
 $n = 100$, left side = $1.03403776602996e-2$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $2.94703393950501e-2$
 $n = 200$, left side = $5.03665536910569e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $1.94697155778688e-2$
 $n = 500$, left side = $1.98226000171708e-3$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.09216402412472e-2$

 $x_0 = 1/4\pi$, Power = 7/10, lamda = 1/4, q = 1

$\sin(x)$

```

n = 10 , left side = 4.55658979518567e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.53960333545031e-1
n = 20 , left side = 1.17886898348851e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.11034112776694e-1
n = 50 , left side = 1.90462319911511e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.27680774586206e-2
n = 100 , left side = 4.76820716817761e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.93338963385320e-2
n = 200 , left side = 1.19246801639550e-4
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.43871241453350e-2
n = 500 , left side = 1.90813536956602e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28848188892687e-2

```

$\cos(x)$

```

n = 10 , left side = 4.55601836086842e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.53966047888204e-1
n = 20 , left side = 1.17886897702854e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.11034112841294e-1
n = 50 , left side = 1.90462319911577e-3

```

```

1/n^(7/10) = 6.46727006577358e-2
difference = 6.27680774586200e-2
n = 100 , left side = 4.76820716817428e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.93338963385323e-2
n = 200 , left side = 1.19246801639439e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.43871241453351e-2
n = 500 , left side = 1.90813536956602e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28848188892687e-2

```

 $x_0 = 1/4\pi$, Power = 7/10, lamda = 1/2, q = 1/4

```

sin(x)
n = 10 , left side = 7.66430508778090e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.22883180619079e-1
n = 20 , left side = 4.38693273663759e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 7.89534752452031e-2
n = 50 , left side = 1.88047297835471e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 4.58679708741887e-2
n = 100 , left side = 9.60447473250947e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.02062423228403e-2
n = 200 , left side = 4.85201986681627e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.96543510801582e-2
n = 500 , left side = 1.95265780521114e-3
1/n^(7/10) = 1.29039002429643e-2

```

difference = 1.09512424377532e-2

$\cos(x)$

n = 10 , left side = 1.15283010872480e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 8.42432206244079e-2
n = 20 , left side = 5.36357377250527e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 6.91870648865263e-2
n = 50 , left side = 2.03721756047669e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 4.43005250529689e-2
n = 100 , left side = 9.99650884011438e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 2.98142082152354e-2
n = 200 , left side = 4.95003918172598e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.95563317652485e-2
n = 500 , left side = 1.96834138039348e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.09355588625708e-2

x0 = 1/4*pi, Power = 7/10, lamda = 1/2, q = 1/2

$\sin(x)$

n = 10 , left side = 3.37717704676072e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.65754461029281e-1
n = 20 , left side = 2.07753148869365e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.02047487724643e-1

```

n = 50 , left side = 9.21511967883226e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.54575809789035e-2
n = 100 , left side = 4.75529633802174e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.50554207173280e-2
n = 200 , left side = 2.41425898359460e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.20921119633799e-2
n = 500 , left side = 9.74447341997586e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.19294529009667e-2

```

$\cos(x)$

```

n = 10 , left side = 6.24222476166085e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.37103983880279e-1
n = 20 , left side = 2.80065121964507e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 9.48162904151284e-2
n = 50 , left side = 1.03752156077472e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.42974850499885e-2
n = 100 , left side = 5.04543146629388e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.47652855890559e-2
n = 200 , left side = 2.48679971416887e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.20195712328056e-2
n = 500 , left side = 9.86054169944683e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.19178460730196e-2

```

$x_0 = 1/4\pi$, Power = 7/10, lamda = 1/2, q = 1

$\sin(x)$

n = 10 , left side = 1.26576588370484e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.86868572659840e-1
n = 20 , left side = 3.19289527212985e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.19629907339449e-1
n = 50 , left side = 5.12151444058917e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.41605492136768e-2
n = 100 , left side = 1.28084179052301e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.96826328762974e-2
n = 200 , left side = 3.20238618125579e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44743470851619e-2
n = 500 , left side = 5.12400106722488e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28987762418971e-2

$\cos(x)$

n = 10 , left side = 1.26576548275704e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.86868576669318e-1
n = 20 , left side = 3.19289294996883e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.19629909661610e-1
n = 50 , left side = 5.12152389308573e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.41605482684272e-2
n = 100 , left side = 1.28084061290612e-4

```

1/n^(7/10) = 3.98107170553497e-2
difference = 3.96826329940591e-2
n = 200 , left side = 3.20239758674346e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44743469711071e-2
n = 500 , left side = 5.12391080254027e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28987763321618e-2

```

```

x0 = 1/4*pi, Power = 7/10, lamda = 1, q = 1/4

```

$\sin(x)$

```

n = 10 , left side = 4.29282034779491e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.56598028018939e-1
n = 20 , left side = 2.30286683697076e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 9.97941342418714e-2
n = 50 , left side = 9.56745400404757e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.51052466536882e-2
n = 100 , left side = 4.84538571465232e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.49653313406974e-2
n = 200 , left side = 2.43500798502960e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.20713629619449e-2
n = 500 , left side = 9.78230233303456e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.19256700096609e-2

```

$\cos(x)$

```

n = 10 , left side = 5.44419628451549e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.45084268651733e-1
n = 20 , left side = 2.59171447278922e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 9.69056578836869e-2
n = 50 , left side = 1.00304303028137e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.46422703549221e-2
n = 100 , left side = 4.96112082798739e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.48495962273623e-2
n = 200 , left side = 2.46392751821078e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.20424434287637e-2
n = 500 , left side = 9.82856796759912e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.19210434462044e-2

```

 $x_0 = 1/4\pi$, Power = 7/10, lamda = 1, q = 1/2

$\sin(x)$

```

n = 10 , left side = 1.98858419604222e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.79640389536466e-1
n = 20 , left side = 1.11111242904842e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.11711678321095e-1
n = 50 , left side = 4.71676040228819e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.99559402554476e-2
n = 100 , left side = 2.40386308996299e-3

```

```

1/n^(7/10) = 3.98107170553497e-2
difference = 3.74068539653867e-2
n = 200 , left side = 1.21454789680442e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.32918230501701e-2
n = 500 , left side = 4.88452065581013e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.24154481773833e-2

```

$\cos(x)$

```

n = 10 , left side = 2.88791443525872e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.70647087144301e-1
n = 20 , left side = 1.33664407456531e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.09456361865926e-1
n = 50 , left side = 5.07733367488394e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.95953669828518e-2
n = 100 , left side = 2.49412468567678e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.73165923696730e-2
n = 200 , left side = 1.23708851068816e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.32692824362863e-2
n = 500 , left side = 4.92063839226731e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.24118364037376e-2

```

$x_0 = 1/4\pi$, Power = 7/10, lamda = 1, q = 1

$\sin(x)$

$n = 10$, left side = 4.09324613380091e-3
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = 1.95432985363087e-1
 $n = 20$, left side = 1.03161560946519e-3
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = 1.21791187002114e-1
 $n = 50$, left side = 1.58852765311801e-4
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = 6.45138478924240e-2
 $n = 100$, left side = 4.13962851463223e-5
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = 3.97693207702034e-2
 $n = 200$, left side = 9.67248590255654e-6
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = 2.44966984610719e-2
 $n = 500$, left side = 2.07035881660822e-6
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = 1.29018298841477e-2

$$\cos(x)$$

$n = 10$, left side = 4.05635916216029e-3
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = 1.95469872334728e-1
 $n = 20$, left side = 1.00937393504252e-3
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = 1.21813428676537e-1
 $n = 50$, left side = 1.67976724238628e-4
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = 6.45047239334971e-2
 $n = 100$, left side = 4.02584365390979e-5
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = 3.97704586188106e-2
 $n = 200$, left side = 1.07750350520908e-5
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = 2.44955959119224e-2

```

n = 500 , left side = 1.19782555663139e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.29027024174077e-2

```

```

-----
x0 = 1/2*pi, Power = 3/10, lamda = 1/4,  q = 1/4
-----

```

$\sin(x)$

```

n = 10 , left side = 9.84826134043602e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.02704620222912e-1
n = 20 , left side = 2.61046861206299e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.80985845416275e-1
n = 50 , left side = 4.22645955857948e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.05023035152412e-1
n = 100 , left side = 1.05840494480935e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50130238206149e-1
n = 200 , left side = 2.64713303538500e-4
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03763864033298e-1
n = 500 , left side = 4.23591513531996e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54949539603481e-1

```

$\cos(x)$

```

n = 10 , left side = 2.54999238457694e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 2.46187995169579e-1
n = 20 , left side = 1.35881405538451e-1

```

```

1/n^(3/10) = 4.07090531536904e-1
difference = 2.71209125998454e-1
n = 50 , left side = 5.52740754515297e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.53975419259462e-1
n = 100 , left side = 2.77036411956434e-2
1/n^(3/10) = 2.51188643150958e-1
difference = 2.23485001955315e-1
n = 200 , left side = 1.38601618045097e-2
1/n^(3/10) = 2.04028577336837e-1
difference = 1.90168415532327e-1
n = 500 , left side = 5.54499938996761e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.49446899364866e-1

```

 $x_0 = 1/2\pi$, Power = 3/10, lamda = 1/4, q = 1/2

```

sin(x)

n = 10 , left side = 7.25561006486467e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.28631132978626e-1
n = 20 , left side = 1.90326036120039e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.88057927924901e-1
n = 50 , left side = 3.07684651802387e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.06172648192968e-1
n = 100 , left side = 7.70350593050750e-4
1/n^(3/10) = 2.51188643150958e-1
difference = 2.50418292557907e-1
n = 200 , left side = 1.92658947488322e-4
1/n^(3/10) = 2.04028577336837e-1

```

```

        difference = 2.03835918389349e-1
n = 500 , left side = 3.08286271140101e-5
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54961070127720e-1

```

$\cos(x)$

```

n = 10 , left side = 1.28642490007521e-1
        1/n^(3/10) = 5.01187233627272e-1
        difference = 3.72544743619751e-1
n = 20 , left side = 6.81042176000301e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.38986313936874e-1
n = 50 , left side = 2.76476637677382e-2
        1/n^(3/10) = 3.09249494710992e-1
        difference = 2.81601830943254e-1
n = 100 , left side = 1.38531517342038e-2
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.37335491416754e-1
n = 200 , left side = 6.93024738450046e-3
        1/n^(3/10) = 2.04028577336837e-1
        difference = 1.97098329952337e-1
n = 500 , left side = 2.77251035146458e-3
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.52219388403369e-1

```

$x_0 = 1/2\pi$, Power = 3/10, lamda = 1/4, q = 1

$\sin(x)$

```

n = 10 , left side = 6.40115086492427e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.37175724978030e-1

```



```

n = 20 , left side = 1.66715291654219e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.90419002371483e-1
n = 50 , left side = 2.69354395939936e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.06555950751592e-1
n = 100 , left side = 6.74326324543451e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50514316826415e-1
n = 200 , left side = 1.68640444147772e-4
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03859936892689e-1
n = 500 , left side = 2.69851091849738e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54964913645649e-1

```

$\cos(x)$

```

n = 10 , left side = 3.68331709595389e-4
          1/n^(3/10) = 5.01187233627272e-1
          difference = 5.00818901917677e-1
n = 20 , left side = 1.81936348617625e-7
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.07090349600556e-1
n = 50 , left side = 0.000000000000000e0
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.09249494710992e-1
n = 100 , left side = 4.77112203438756e-17
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.51188643150958e-1
n = 200 , left side = 3.99794287396218e-18
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.04028577336837e-1
n = 500 , left side = 8.20395785813390e-17
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54991898754834e-1

```

 $x_0 = 1/2\pi$, Power = 3/10, lamda = 1/2, q = 1/4

$\sin(x)$

```

n = 10 , left side = 2.73218987411665e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.73865334886106e-1
n = 20 , left side = 6.90589468087810e-3
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.00184636856026e-1
n = 50 , left side = 1.10835163775858e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.08141143073233e-1
n = 100 , left side = 2.77209959256952e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50911433191701e-1
n = 200 , left side = 6.93101217303926e-5
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03959267215107e-1
n = 500 , left side = 1.10899618147586e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54980808793019e-1

```

$\cos(x)$

```

n = 10 , left side = 1.35711685154782e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.65475548472490e-1
n = 20 , left side = 6.89464901817200e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.38144041355184e-1
n = 50 , left side = 2.77022557225578e-2

```

```

1/n^(3/10) = 3.09249494710992e-1
difference = 2.81547238988434e-1
n = 100 , left side = 1.38599886244430e-2
1/n^(3/10) = 2.51188643150958e-1
difference = 2.37328654526515e-1
n = 200 , left side = 6.93110233963076e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.97097474997206e-1
n = 500 , left side = 2.77256501313650e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.52219333741697e-1

```

 $x_0 = 1/2\pi$, Power = 3/10, lamda = 1/2, q = 1/2

$\sin(x)$

```

n = 10 , left side = 2.02586050377558e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.80928628589517e-1
n = 20 , left side = 5.11322907698697e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.01977302459917e-1
n = 50 , left side = 8.20311672977692e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.08429183038014e-1
n = 100 , left side = 2.05156512414151e-4
1/n^(3/10) = 2.51188643150958e-1
difference = 2.50983486638544e-1
n = 200 , left side = 5.12940438202047e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03977283293017e-1
n = 500 , left side = 8.20726742556399e-6
1/n^(3/10) = 1.54991898754834e-1

```

difference = 1.54983691487408e-1

$\cos(x)$

n = 10 , left side = 6.80191579414196e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.33168075685853e-1
n = 20 , left side = 3.44939608010511e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.72596570735853e-1
n = 50 , left side = 1.38524597520625e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.95397034958929e-1
n = 100 , left side = 6.93016042367867e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.44258482727279e-1
n = 200 , left side = 3.46557187207862e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.00563005464758e-1
n = 500 , left side = 1.38628390798491e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53605614846849e-1

x0 = 1/2*pi, Power = 3/10, lamda = 1/2, q = 1

$\sin(x)$

n = 10 , left side = 1.79004576579427e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.83286775969330e-1
n = 20 , left side = 4.51543399532361e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.02575097541581e-1

```

n = 50 , left side = 7.24292150887096e-4
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.08525202560105e-1
n = 100 , left side = 1.81138319373497e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.51007504831585e-1
n = 200 , left side = 4.52886594521917e-5
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03983288677385e-1
n = 500 , left side = 7.24636777071908e-6
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54984652387063e-1

```

$\cos(x)$

```

n = 10 , left side = 1.57869523831611e-7
          1/n^(3/10) = 5.01187233627272e-1
          difference = 5.01187075757749e-1
n = 20 , left side = 8.57307872321585e-10
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.07090530679597e-1
n = 50 , left side = 1.66778292572710e-10
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.09249494544213e-1
n = 100 , left side = 1.61355749321499e-10
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.51188642989602e-1
n = 200 , left side = 1.41525527114055e-10
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.04028577195311e-1
n = 500 , left side = 4.01468831931699e-11
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54991898714687e-1

```

$x_0 = 1/2\pi$, Power = 3/10, lamda = 1, q = 1/4

$\sin(x)$

```
n = 10 , left side = 8.14417705162507e-3
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.93043056575647e-1
n = 20 , left side = 2.04451572215358e-3
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.05046015814751e-1
n = 50 , left side = 3.27307123230036e-4
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.08922187587762e-1
n = 100 , left side = 8.17942128127358e-5
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.51106848938145e-1
n = 200 , left side = 2.04561035900275e-5
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.04008121233247e-1
n = 500 , left side = 3.27470541416908e-6
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54988624049420e-1
```

$\cos(x)$

```
n = 10 , left side = 6.88796875744184e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.32307546052854e-1
n = 20 , left side = 3.46106749496968e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.72479856587208e-1
n = 50 , left side = 1.38657599904077e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.95383734720584e-1
n = 100 , left side = 6.92779875289445e-3
```

```

1/n^(3/10) = 2.51188643150958e-1
difference = 2.44260844398064e-1
n = 200 , left side = 3.46437879098605e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.00564198545851e-1
n = 500 , left side = 1.38664352198994e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53605255232844e-1

```

 $x_0 = 1/2\pi$, Power = 3/10, lamda = 1, q = 1/2

$\sin(x)$

```

n = 10 , left side = 6.36177783107461e-3
1/n^(3/10) = 5.01187233627272e-1
difference = 4.94825455796198e-1
n = 20 , left side = 1.59327072888782e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.05497260808017e-1
n = 50 , left side = 2.55270681782438e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.08994224029209e-1
n = 100 , left side = 6.37527543482230e-5
1/n^(3/10) = 2.51188643150958e-1
difference = 2.51124890396610e-1
n = 200 , left side = 1.59359198846909e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.04012641416952e-1
n = 500 , left side = 2.55130565363082e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54989347449180e-1

```

$\cos(x)$

```

n = 10 , left side = 3.44617147652922e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.66725518861980e-1
n = 20 , left side = 1.72865571687706e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.89803974368134e-1
n = 50 , left side = 6.92555552990447e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.02323939181087e-1
n = 100 , left side = 3.46698227542804e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.47721660875530e-1
n = 200 , left side = 1.73278247468734e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.02295794862150e-1
n = 500 , left side = 6.92495905051582e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54299402849782e-1

```

 $x_0 = 1/2\pi$, Power = 3/10, lamda = 1, q = 1

$\sin(x)$

```

n = 10 , left side = 5.75819452524406e-3
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.95429039102028e-1
n = 20 , left side = 1.44242808346062e-3
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.05648103453444e-1
n = 50 , left side = 2.30930988249112e-4
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.09018563722743e-1
n = 100 , left side = 5.78328593896504e-5

```



```

1/n^(3/10) = 2.51188643150958e-1
difference = 2.51130810291568e-1
n = 200 , left side = 1.44542703056638e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.04014123066531e-1
n = 500 , left side = 2.30995962646752e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54989588795207e-1

```

$\cos(x)$

```

n = 10 , left side = 3.17056957943215e-5
1/n^(3/10) = 5.01187233627272e-1
difference = 5.01155527931478e-1
n = 20 , left side = 8.21112944009224e-6
1/n^(3/10) = 4.07090531536904e-1
difference = 4.07082320407464e-1
n = 50 , left side = 1.60967600009258e-6
1/n^(3/10) = 3.09249494710992e-1
difference = 3.09247885034992e-1
n = 100 , left side = 1.55936473946361e-6
1/n^(3/10) = 2.51188643150958e-1
difference = 2.51187083786219e-1
n = 200 , left side = 1.36805461285126e-6
1/n^(3/10) = 2.04028577336837e-1
difference = 2.04027209282224e-1
n = 500 , left side = 3.88056297516311e-7
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54991510698536e-1

```

$x_0 = 1/2\pi$, Power = 1/2, lamda = 1/4, q = 1/4

$\sin(x)$

$n = 10$, left side = $9.84826134043602e-2$
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = $2.17745152612478e-1$
 $n = 20$, left side = $2.61046861206299e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $1.97502111629349e-1$
 $n = 50$, left side = $4.22645955857948e-3$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $1.37194896678730e-1$
 $n = 100$, left side = $1.05840494480935e-3$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $9.89415950551907e-2$
 $n = 200$, left side = $2.64713303538500e-4$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $7.04459648151163e-2$
 $n = 500$, left side = $4.23591513531996e-5$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.46790003986426e-2$

$$\cos(x)$$

$n = 10$, left side = $2.54999238457694e-1$
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = $6.12285275591444e-2$
 $n = 20$, left side = $1.35881405538451e-1$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $8.77253922115283e-2$
 $n = 50$, left side = $5.52740754515297e-2$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $8.61472807857798e-2$
 $n = 100$, left side = $2.77036411956434e-2$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $7.22963588043566e-2$
 $n = 200$, left side = $1.38601618045097e-2$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $5.68505163141450e-2$

```

n = 500 , left side = 5.54499938996761e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 3.91763601600282e-2

```

```

-----
x0 = 1/2*pi, Power = 1/2, lamda = 1/4,  q = 1/2
-----

```

$\sin(x)$

```

n = 10 , left side = 7.25561006486467e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.43671665368191e-1
n = 20 , left side = 1.90326036120039e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.04574194137975e-1
n = 50 , left side = 3.07684651802387e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.38344509719286e-1
n = 100 , left side = 7.70350593050750e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.92296494069493e-2
n = 200 , left side = 1.92658947488322e-4
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.05180191711664e-2
n = 500 , left side = 3.08286271140101e-5
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.46905309228818e-2

```

$\cos(x)$

```

n = 10 , left side = 1.28642490007521e-1
          1/n^(1/2) = 3.16227766016838e-1
          difference = 1.87585276009317e-1
n = 20 , left side = 6.81042176000301e-2

```

```

1/n^(1/2) = 2.23606797749979e-1
difference = 1.55502580149949e-1
n = 50 , left side = 2.76476637677382e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.13773692469571e-1
n = 100 , left side = 1.38531517342038e-2
1/n^(1/2) = 1.00000000000000e-1
difference = 8.61468482657962e-2
n = 200 , left side = 6.93024738450046e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.37804307341543e-2
n = 500 , left side = 2.77251035146458e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.19488491985312e-2

```

x0 = 1/2*pi, Power = 1/2, lamda = 1/4, q = 1

```

sin(x)
n = 10 , left side = 6.40115086492427e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.52216257367595e-1
n = 20 , left side = 1.66715291654219e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.06935268584557e-1
n = 50 , left side = 2.69354395939936e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.38727812277910e-1
n = 100 , left side = 6.74326324543451e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.93256736754566e-2
n = 200 , left side = 1.68640444147772e-4
1/n^(1/2) = 7.07106781186548e-2

```

```

difference = 7.05420376745070e-2
n = 500 , left side = 2.69851091849738e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46943744408108e-2

```

$\cos(x)$

```

n = 10 , left side = 3.68331709595389e-4
1/n^(1/2) = 3.16227766016838e-1
difference = 3.15859434307243e-1
n = 20 , left side = 1.81936348617625e-7
1/n^(1/2) = 2.23606797749979e-1
difference = 2.23606615813630e-1
n = 50 , left side = 0.000000000000000e0
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41421356237310e-1
n = 100 , left side = 4.77112203438756e-17
1/n^(1/2) = 1.000000000000000e-1
difference = 1.000000000000000e-1
n = 200 , left side = 3.99794287396218e-18
1/n^(1/2) = 7.07106781186548e-2
difference = 7.07106781186548e-2
n = 500 , left side = 8.20395785813390e-17
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47213595499957e-2

```

$x_0 = 1/2\pi$, Power = 1/2, lamda = 1/2, q = 1/4

$\sin(x)$

```

n = 10 , left side = 2.73218987411665e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.88905867275671e-1

```

$n = 20$, left side = $6.90589468087810e-3$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $2.16700903069101e-1$
 $n = 50$, left side = $1.10835163775858e-3$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $1.40313004599551e-1$
 $n = 100$, left side = $2.77209959256952e-4$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $9.97227900407431e-2$
 $n = 200$, left side = $6.93101217303926e-5$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $7.06413679969244e-2$
 $n = 500$, left side = $1.10899618147586e-5$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.47102695881810e-2$

$\cos(x)$

$n = 10$, left side = $1.35711685154782e-1$
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = $1.80516080862056e-1$
 $n = 20$, left side = $6.89464901817200e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $1.54660307568259e-1$
 $n = 50$, left side = $2.77022557225578e-2$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $1.13719100514752e-1$
 $n = 100$, left side = $1.38599886244430e-2$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $8.61400113755570e-2$
 $n = 200$, left side = $6.93110233963076e-3$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $6.37795757790240e-2$
 $n = 500$, left side = $2.77256501313650e-3$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.19487945368593e-2$

 $x_0 = 1/2\pi$, Power = 1/2, lamda = 1/2, q = 1/2

$\sin(x)$

n = 10 , left side = 2.02586050377558e-2
 1/n^(1/2) = 3.16227766016838e-1
 difference = 2.95969160979082e-1
 n = 20 , left side = 5.11322907698697e-3
 1/n^(1/2) = 2.23606797749979e-1
 difference = 2.18493568672992e-1
 n = 50 , left side = 8.20311672977692e-4
 1/n^(1/2) = 1.41421356237310e-1
 difference = 1.40601044564332e-1
 n = 100 , left side = 2.05156512414151e-4
 1/n^(1/2) = 1.00000000000000e-1
 difference = 9.97948434875859e-2
 n = 200 , left side = 5.12940438202047e-5
 1/n^(1/2) = 7.07106781186548e-2
 difference = 7.06593840748345e-2
 n = 500 , left side = 8.20726742556399e-6
 1/n^(1/2) = 4.47213595499958e-2
 difference = 4.47131522825702e-2

$\cos(x)$

n = 10 , left side = 6.80191579414196e-2
 1/n^(1/2) = 3.16227766016838e-1
 difference = 2.48208608075418e-1
 n = 20 , left side = 3.44939608010511e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 1.89112836948928e-1
 n = 50 , left side = 1.38524597520625e-2

```

1/n^(1/2) = 1.41421356237310e-1
difference = 1.27568896485247e-1
n = 100 , left side = 6.93016042367867e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.30698395763213e-2
n = 200 , left side = 3.46557187207862e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.72451062465761e-2
n = 500 , left side = 1.38628390798491e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.33350756420109e-2

```

 $x_0 = 1/2\pi$, Power = 1/2, lamda = 1/2, q = 1

```

sin(x)
n = 10 , left side = 1.79004576579427e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.98327308358895e-1
n = 20 , left side = 4.51543399532361e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.19091363754655e-1
n = 50 , left side = 7.24292150887096e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.40697064086422e-1
n = 100 , left side = 1.81138319373497e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.98188616806265e-2
n = 200 , left side = 4.52886594521917e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06653894592026e-2
n = 500 , left side = 7.24636777071908e-6
1/n^(1/2) = 4.47213595499958e-2

```


difference = 4.47141131822251e-2

$\cos(x)$

n = 10 , left side = 1.57869523831611e-7
1/n^(1/2) = 3.16227766016838e-1
difference = 3.16227608147314e-1
n = 20 , left side = 8.57307872321585e-10
1/n^(1/2) = 2.23606797749979e-1
difference = 2.23606796892671e-1
n = 50 , left side = 1.66778292572710e-10
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41421356070531e-1
n = 100 , left side = 1.61355749321499e-10
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99999998386443e-2
n = 200 , left side = 1.41525527114055e-10
1/n^(1/2) = 7.07106781186548e-2
difference = 7.07106779771292e-2
n = 500 , left side = 4.01468831931699e-11
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47213595098489e-2

x0 = 1/2*pi, Power = 1/2, lamda = 1, q = 1/4

$\sin(x)$

n = 10 , left side = 8.14417705162507e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.08083588965213e-1
n = 20 , left side = 2.04451572215358e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.21562282027825e-1

```

n = 50 , left side = 3.27307123230036e-4
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.41094049114079e-1
n = 100 , left side = 8.17942128127358e-5
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.99182057871873e-2
n = 200 , left side = 2.04561035900275e-5
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.06902220150647e-2
n = 500 , left side = 3.27470541416908e-6
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47180848445816e-2

```

$\cos(x)$

```

n = 10 , left side = 6.88796875744184e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.47348078442420e-1
n = 20 , left side = 3.46106749496968e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.88996122800282e-1
n = 50 , left side = 1.38657599904077e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.27555596246902e-1
n = 100 , left side = 6.92779875289445e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.30722012471056e-2
n = 200 , left side = 3.46437879098605e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.72462993276687e-2
n = 500 , left side = 1.38664352198994e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.33347160280059e-2

```

$x_0 = 1/2\pi$, Power = 1/2, lamda = 1, q = 1/2

$\sin(x)$

n = 10 , left side = 6.36177783107461e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.09865988185763e-1
n = 20 , left side = 1.59327072888782e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.22013527021091e-1
n = 50 , left side = 2.55270681782438e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41166085555527e-1
n = 100 , left side = 6.37527543482230e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99362472456518e-2
n = 200 , left side = 1.59359198846909e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06947421987701e-2
n = 500 , left side = 2.55130565363082e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47188082443422e-2

$\cos(x)$

n = 10 , left side = 3.44617147652922e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.81766051251546e-1
n = 20 , left side = 1.72865571687706e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.06320240581208e-1
n = 50 , left side = 6.92555552990447e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.34495800707405e-1
n = 100 , left side = 3.46698227542804e-3

```

1/n^(1/2) = 1.000000000000000e-1
difference = 9.65330177245720e-2
n = 200 , left side = 1.73278247468734e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.89778956439674e-2
n = 500 , left side = 6.92495905051582e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.40288636449442e-2

```

 $x_0 = 1/2\pi$, Power = 1/2, lamda = 1, q = 1

$\sin(x)$

```

n = 10 , left side = 5.75819452524406e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.10469571491594e-1
n = 20 , left side = 1.44242808346062e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.22164369666518e-1
n = 50 , left side = 2.30930988249112e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41190425249060e-1
n = 100 , left side = 5.78328593896504e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99421671406104e-2
n = 200 , left side = 1.44542703056638e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06962238483491e-2
n = 500 , left side = 2.30995962646752e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47190495903693e-2

```

$\cos(x)$

```

n = 10 , left side = 3.17056957943215e-5
          1/n^(1/2) = 3.16227766016838e-1
          difference = 3.16196060321044e-1
n = 20 , left side = 8.21112944009224e-6
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.23598586620539e-1
n = 50 , left side = 1.60967600009258e-6
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.41419746561309e-1
n = 100 , left side = 1.55936473946361e-6
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.99984406352605e-2
n = 200 , left side = 1.36805461285126e-6
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.07093100640419e-2
n = 500 , left side = 3.88056297516311e-7
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47209714936983e-2

```

 $x_0 = 1/2\pi$, Power = 7/10, lamda = 1/4, q = 1/4

$\sin(x)$

```

n = 10 , left side = 9.84826134043602e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.01043618092528e-1
n = 20 , left side = 2.61046861206299e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 9.67181164909492e-2
n = 50 , left side = 4.22645955857948e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.04462410991563e-2
n = 100 , left side = 1.05840494480935e-3

```

```

1/n^(7/10) = 3.98107170553497e-2
difference = 3.87523121105404e-2
n = 200 , left side = 2.64713303538500e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.42416576434360e-2
n = 500 , left side = 4.23591513531996e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28615410916111e-2

```

$\cos(x)$

```

n = 10 , left side = 2.54999238457694e-1
1/n^(7/10) = 1.99526231496888e-1
difference = -5.54730069608056e-2
n = 20 , left side = 1.35881405538451e-1
1/n^(7/10) = 1.22822802611579e-1
difference = -1.30586029268716e-2
n = 50 , left side = 5.52740754515297e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 9.39862520620602e-3
n = 100 , left side = 2.77036411956434e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 1.21070758597063e-2
n = 200 , left side = 1.38601618045097e-2
1/n^(7/10) = 2.45063709469745e-2
difference = 1.06462091424648e-2
n = 500 , left side = 5.54499938996761e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 7.35890085299671e-3

```

$x_0 = 1/2\pi$, Power = 7/10, lamda = 1/4, q = 1/2

$\sin(x)$

$n = 10$, left side = $7.25561006486467e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.26970130848241e-1$
 $n = 20$, left side = $1.90326036120039e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $1.03790198999575e-1$
 $n = 50$, left side = $3.07684651802387e-3$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $6.15958541397119e-2$
 $n = 100$, left side = $7.70350593050750e-4$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.90403664622990e-2$
 $n = 200$, left side = $1.92658947488322e-4$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.43137119994862e-2$
 $n = 500$, left side = $3.08286271140101e-5$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.28730716158503e-2$

$\cos(x)$

$n = 10$, left side = $1.28642490007521e-1$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $7.08837414893667e-2$
 $n = 20$, left side = $6.81042176000301e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $5.47185850115489e-2$
 $n = 50$, left side = $2.76476637677382e-2$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $3.70250368899976e-2$
 $n = 100$, left side = $1.38531517342038e-2$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $2.59575653211459e-2$
 $n = 200$, left side = $6.93024738450046e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $1.75761235624740e-2$

```

n = 500 , left side = 2.77251035146458e-3
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.01313898914997e-2

```

```

-----
x0 = 1/2*pi, Power = 7/10, lamda = 1/4,  q = 1
-----

```

$\sin(x)$

```

n = 10 , left side = 6.40115086492427e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.35514722847645e-1
n = 20 , left side = 1.66715291654219e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.06151273446157e-1
n = 50 , left side = 2.69354395939936e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.19791566983364e-2
n = 100 , left side = 6.74326324543451e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.91363907308063e-2
n = 200 , left side = 1.68640444147772e-4
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.43377305028267e-2
n = 500 , left side = 2.69851091849738e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28769151337793e-2

```

$\cos(x)$

```

n = 10 , left side = 3.68331709595389e-4
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.99157899787293e-1
n = 20 , left side = 1.81936348617625e-7

```



```

1/n^(7/10) = 1.22822802611579e-1
difference = 1.22822620675230e-1
n = 50 , left side = 0.000000000000000e0
1/n^(7/10) = 6.46727006577358e-2
difference = 6.46727006577358e-2
n = 100 , left side = 4.77112203438756e-17
1/n^(7/10) = 3.98107170553497e-2
difference = 3.98107170553497e-2
n = 200 , left side = 3.99794287396218e-18
1/n^(7/10) = 2.45063709469745e-2
difference = 2.45063709469745e-2
n = 500 , left side = 8.20395785813390e-17
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29039002429642e-2

```

 $x_0 = 1/2\pi$, Power = 7/10, lamda = 1/2, q = 1/4

```

sin(x)

n = 10 , left side = 2.73218987411665e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.72204332755721e-1
n = 20 , left side = 6.90589468087810e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.15916907930701e-1
n = 50 , left side = 1.10835163775858e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 6.35643490199772e-2
n = 100 , left side = 2.77209959256952e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.95335070960928e-2
n = 200 , left side = 6.93101217303926e-5
1/n^(7/10) = 2.45063709469745e-2

```

```

difference = 2.44370608252441e-2
n = 500 , left side = 1.10899618147586e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28928102811496e-2

```

$\cos(x)$

```

n = 10 , left side = 1.35711685154782e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 6.38145463421061e-2
n = 20 , left side = 6.89464901817200e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 5.38763124298590e-2
n = 50 , left side = 2.77022557225578e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 3.69704449351780e-2
n = 100 , left side = 1.38599886244430e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 2.59507284309068e-2
n = 200 , left side = 6.93110233963076e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.75752686073437e-2
n = 500 , left side = 2.77256501313650e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.01313352298278e-2

```

 $x_0 = 1/2\pi$, Power = 7/10, lamda = 1/2, q = 1/2

$\sin(x)$

```

n = 10 , left side = 2.02586050377558e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.79267626459132e-1

```

$n = 20$, left side = $5.11322907698697e-3$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $1.17709573534592e-1$
 $n = 50$, left side = $8.20311672977692e-4$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $6.38523889847581e-2$
 $n = 100$, left side = $2.05156512414151e-4$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.96055605429356e-2$
 $n = 200$, left side = $5.12940438202047e-5$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.44550769031543e-2$
 $n = 500$, left side = $8.20726742556399e-6$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.28956929755388e-2$

$$\cos(x)$$

$n = 10$, left side = $6.80191579414196e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.31507073555468e-1$
 $n = 20$, left side = $3.44939608010511e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $8.83288418105280e-2$
 $n = 50$, left side = $1.38524597520625e-2$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $5.08202409056733e-2$
 $n = 100$, left side = $6.93016042367867e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.28805566316711e-2$
 $n = 200$, left side = $3.46557187207862e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.10407990748959e-2$
 $n = 500$, left side = $1.38628390798491e-3$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.15176163349794e-2$

 $x_0 = 1/2\pi$, Power = 7/10, lamda = 1/2, q = 1

$\sin(x)$

```

n = 10 , left side = 1.79004576579427e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.81625773838945e-1
n = 20 , left side = 4.51543399532361e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.18307368616255e-1
n = 50 , left side = 7.24292150887096e-4
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.39484085068487e-2
n = 100 , left side = 1.81138319373497e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.96295787359762e-2
n = 200 , left side = 4.52886594521917e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44610822875223e-2
n = 500 , left side = 7.24636777071908e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28966538751936e-2

```

$\cos(x)$

```

n = 10 , left side = 1.57869523831611e-7
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.99526073627364e-1
n = 20 , left side = 8.57307872321585e-10
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.22822801754271e-1
n = 50 , left side = 1.66778292572710e-10

```

```

1/n^(7/10) = 6.46727006577358e-2
difference = 6.46727004909575e-2
n = 100 , left side = 1.61355749321499e-10
1/n^(7/10) = 3.98107170553497e-2
difference = 3.98107168939940e-2
n = 200 , left side = 1.41525527114055e-10
1/n^(7/10) = 2.45063709469745e-2
difference = 2.45063708054490e-2
n = 500 , left side = 4.01468831931699e-11
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29039002028174e-2

```

 $x_0 = 1/2\pi$, Power = 7/10, lamda = 1, q = 1/4

$\sin(x)$

```

n = 10 , left side = 8.14417705162507e-3
1/n^(7/10) = 1.99526231496888e-1
difference = 1.91382054445263e-1
n = 20 , left side = 2.04451572215358e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.20778286889425e-1
n = 50 , left side = 3.27307123230036e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.43453935345057e-2
n = 100 , left side = 8.17942128127358e-5
1/n^(7/10) = 3.98107170553497e-2
difference = 3.97289228425370e-2
n = 200 , left side = 2.04561035900275e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44859148433845e-2
n = 500 , left side = 3.27470541416908e-6
1/n^(7/10) = 1.29039002429643e-2

```

difference = 1.29006255375502e-2

$\cos(x)$

n = 10 , left side = 6.88796875744184e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.30646543922470e-1
n = 20 , left side = 3.46106749496968e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 8.82121276618822e-2
n = 50 , left side = 1.38657599904077e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 5.08069406673280e-2
n = 100 , left side = 6.92779875289445e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.28829183024553e-2
n = 200 , left side = 3.46437879098605e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.10419921559885e-2
n = 500 , left side = 1.38664352198994e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.15172567209744e-2

x0 = 1/2*pi, Power = 7/10, lamda = 1, q = 1/2

$\sin(x)$

n = 10 , left side = 6.36177783107461e-3
1/n^(7/10) = 1.99526231496888e-1
difference = 1.93164453665813e-1
n = 20 , left side = 1.59327072888782e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.21229531882691e-1

```

n = 50 , left side = 2.55270681782438e-4
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.44174299759533e-2
n = 100 , left side = 6.37527543482230e-5
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.97469643010015e-2
n = 200 , left side = 1.59359198846909e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44904350270898e-2
n = 500 , left side = 2.55130565363082e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.29013489373107e-2

```

$\cos(x)$

```

n = 10 , left side = 3.44617147652922e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.65064516731596e-1
n = 20 , left side = 1.72865571687706e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.05536245442808e-1
n = 50 , left side = 6.92555552990447e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.77471451278313e-2
n = 100 , left side = 3.46698227542804e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.63437347799217e-2
n = 200 , left side = 1.73278247468734e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.27735884722872e-2
n = 500 , left side = 6.92495905051582e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.22114043379127e-2

```

$x_0 = 1/2\pi$, Power = 7/10, lamda = 1, q = 1

$\sin(x)$

n = 10 , left side = 5.75819452524406e-3
1/n^(7/10) = 1.99526231496888e-1
difference = 1.93768036971644e-1
n = 20 , left side = 1.44242808346062e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.21380374528118e-1
n = 50 , left side = 2.30930988249112e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.44417696694866e-2
n = 100 , left side = 5.78328593896504e-5
1/n^(7/10) = 3.98107170553497e-2
difference = 3.97528841959601e-2
n = 200 , left side = 1.44542703056638e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44919166766688e-2
n = 500 , left side = 2.30995962646752e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29015902833379e-2

$\cos(x)$

n = 10 , left side = 3.17056957943215e-5
1/n^(7/10) = 1.99526231496888e-1
difference = 1.99494525801094e-1
n = 20 , left side = 8.21112944009224e-6
1/n^(7/10) = 1.22822802611579e-1
difference = 1.22814591482139e-1
n = 50 , left side = 1.60967600009258e-6
1/n^(7/10) = 6.46727006577358e-2
difference = 6.46710909817357e-2
n = 100 , left side = 1.55936473946361e-6


```

1/n^(7/10) = 3.98107170553497e-2
difference = 3.98091576906103e-2
n = 200 , left side = 1.36805461285126e-6
1/n^(7/10) = 2.45063709469745e-2
difference = 2.45050028923617e-2
n = 500 , left side = 3.88056297516311e-7
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29035121866668e-2

```

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1/4, q = 1/4

$\sin(x)$

```

n = 10 , left side = 2.00168857534962e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.01018376092311e-1
n = 20 , left side = 1.13262072722325e-1
1/n^(3/10) = 4.07090531536904e-1
difference = 2.93828458814580e-1
n = 50 , left side = 4.20732246994345e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.67176270011557e-1
n = 100 , left side = 2.03378378667144e-2
1/n^(3/10) = 2.51188643150958e-1
difference = 2.30850805284244e-1
n = 200 , left side = 9.98779497231383e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.94040782364523e-1
n = 500 , left side = 3.95085911348803e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.51041039641346e-1

```

$\cos(x)$

```

n = 10 , left side = 9.93951714238188e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.01792062203454e-1
n = 20 , left side = 7.72328322774641e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.29857699259440e-1
n = 50 , left side = 3.60961128316283e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.73153381879363e-1
n = 100 , left side = 1.88410272392823e-2
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.32347615911676e-1
n = 200 , left side = 9.61343382830904e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.94415143508528e-1
n = 500 , left side = 3.89095422715435e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.51100944527679e-1

```

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1/4, q = 1/2

$\sin(x)$

```

n = 10 , left side = 1.13322309591826e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.87864924035447e-1
n = 20 , left side = 6.09253390188926e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.46165192518012e-1
n = 50 , left side = 2.17255059254714e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.87523988785520e-1
n = 100 , left side = 1.03403776602992e-2

```

```

1/n^(3/10) = 2.51188643150958e-1
difference = 2.40848265490659e-1
n = 200 , left side = 5.03665536910580e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.98991921967731e-1
n = 500 , left side = 1.98226000171664e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53009638753117e-1

```

$\cos(x)$

```

n = 10 , left side = 3.12663766609833e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.69920856966289e-1
n = 20 , left side = 3.44627783783421e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.72627753158562e-1
n = 50 , left side = 1.73741901387926e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.91875304572199e-1
n = 100 , left side = 9.25093740382432e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.41937705747134e-1
n = 200 , left side = 4.76419447265553e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.99264382864181e-1
n = 500 , left side = 1.93866173914425e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53053237015689e-1

```

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1/4, q = 1

$\sin(x)$

$n = 10$, left side = $2.86823828109783e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.72504850816294e-1$
 $n = 20$, left side = $1.14200334403044e-2$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $3.95670498096600e-1$
 $n = 50$, left side = $1.90462132680691e-3$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $3.07344873384185e-1$
 $n = 100$, left side = $4.76820716817317e-4$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.50711822434141e-1$
 $n = 200$, left side = $1.19246801639772e-4$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.03909330535197e-1$
 $n = 500$, left side = $1.90813536952161e-5$
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = $1.54972817401138e-1$

$$\cos(x)$$

$n = 10$, left side = $5.13589058368008e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.49828327790472e-1$
 $n = 20$, left side = $1.19286950374289e-2$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $3.95161836499476e-1$
 $n = 50$, left side = $1.90462392585333e-3$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $3.07344870785138e-1$
 $n = 100$, left side = $4.76820716817428e-4$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.50711822434141e-1$
 $n = 200$, left side = $1.19246801639328e-4$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.03909330535198e-1$

```

n = 500 , left side = 1.90813536953272e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54972817401138e-1

```

```

-----
x0 = 3/4*pi, Power = 3/10, lamda = 1/2,  q = 1/4
-----

```

$\sin(x)$

```

n = 10 , left side = 1.14144646548721e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.87042587078551e-1
n = 20 , left side = 5.36350821680255e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.53455449368879e-1
n = 50 , left side = 2.03721764988826e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.88877318212109e-1
n = 100 , left side = 9.99650895754645e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.41192134193412e-1
n = 200 , left side = 4.95003907205882e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.99078538264778e-1
n = 500 , left side = 1.96834136039681e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.53023557394437e-1

```

$\cos(x)$

```

n = 10 , left side = 7.63132390190502e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.24873994608222e-1
n = 20 , left side = 4.38690929008562e-2

```

```

1/n^(3/10) = 4.07090531536904e-1
difference = 3.63221438636048e-1
n = 50 , left side = 1.88047305777048e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.90444764133287e-1
n = 100 , left side = 9.60447484518001e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.41584168305778e-1
n = 200 , left side = 4.85201975765370e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 1.99176557579183e-1
n = 500 , left side = 1.95265778643006e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53039240968404e-1

```

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1/2, q = 1/2

```

sin(x)

n = 10 , left side = 6.18104658618466e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.39376767765426e-1
n = 20 , left side = 2.80061711992697e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.79084360337635e-1
n = 50 , left side = 1.03752156519760e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.98874279059016e-1
n = 100 , left side = 5.04543144574476e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.46143211705213e-1
n = 200 , left side = 2.48679978426469e-3
1/n^(3/10) = 2.04028577336837e-1

```

```

        difference = 2.01541777552572e-1
n = 500 , left side = 9.86054094334721e-4
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54005844660499e-1

```

$\cos(x)$

```

n = 10 , left side = 3.35723331340523e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.67614900493220e-1
n = 20 , left side = 2.07751849096550e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.86315346627249e-1
n = 50 , left side = 9.21511983819845e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.00034374872793e-1
n = 100 , left side = 4.75529632502636e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.46433346825932e-1
n = 200 , left side = 2.41425905034953e-3
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.01614318286487e-1
n = 500 , left side = 9.74447265982392e-4
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54017451488851e-1

```

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1/2, q = 1

$\sin(x)$

```

n = 10 , left side = 1.23317176677876e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.88855515959485e-1

```

$n = 20$, left side = $3.19271958223932e-3$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $4.03897811954665e-1$
 $n = 50$, left side = $5.12151488168966e-4$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $3.08737343222823e-1$
 $n = 100$, left side = $1.28083959738734e-4$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.51060559191219e-1$
 $n = 200$, left side = $3.20240359172885e-5$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.03996553300920e-1$
 $n = 500$, left side = $5.12398355012600e-6$
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = $1.54986774771284e-1$

$\cos(x)$

$n = 10$, left side = $1.27760703918800e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.88411163235392e-1$
 $n = 20$, left side = $3.19296312784956e-3$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $4.03897568409055e-1$
 $n = 50$, left side = $5.12152374566921e-4$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $3.08737342336425e-1$
 $n = 100$, left side = $1.28084284152230e-4$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.51060558866806e-1$
 $n = 200$, left side = $3.20237987473382e-5$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.03996553538090e-1$
 $n = 500$, left side = $5.12392899754133e-6$
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = $1.54986774825836e-1$

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1, q = 1/4

$\sin(x)$

```

n = 10 , left side = 5.44764920559380e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.46710741571334e-1
n = 20 , left side = 2.59004027714145e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.81190128765490e-1
n = 50 , left side = 1.00311377175122e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.99218356993480e-1
n = 100 , left side = 4.96093970818390e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.46227703442774e-1
n = 200 , left side = 2.46459813024336e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.01563979206594e-1
n = 500 , left side = 9.82124425668585e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54009774329165e-1

```

$\cos(x)$

```

n = 10 , left side = 4.29522822629749e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.58234951364297e-1
n = 20 , left side = 2.30109143999059e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.84079617136999e-1
n = 50 , left side = 9.56870414660826e-3

```

```

1/n^(3/10) = 3.09249494710992e-1
difference = 2.99680790564383e-1
n = 100 , left side = 4.84524222640814e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.46343400924550e-1
n = 200 , left side = 2.43566015728058e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.01592917179556e-1
n = 500 , left side = 9.77496918584397e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54014401836249e-1

```

 $x_0 = 3/4\pi$, Power = 3/10, lamda = 1, q = 1/2

$\sin(x)$

```

n = 10 , left side = 2.88447866653312e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.72342446961941e-1
n = 20 , left side = 1.33629574745268e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.93727574062378e-1
n = 50 , left side = 5.08446410303787e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.04165030607954e-1
n = 100 , left side = 2.49515406149969e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.48693489089458e-1
n = 200 , left side = 1.23598531292535e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.02792592023912e-1
n = 500 , left side = 4.92089942069840e-4
1/n^(3/10) = 1.54991898754834e-1

```

difference = 1.54499808812764e-1

$\cos(x)$

n = 10 , left side = 1.98502218336606e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.81337011793612e-1
n = 20 , left side = 1.11111768757201e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.95979354661184e-1
n = 50 , left side = 4.72342001665071e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.04526074694341e-1
n = 100 , left side = 2.40486942054952e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.48783773730408e-1
n = 200 , left side = 1.21344788187139e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.02815129454966e-1
n = 500 , left side = 4.88483607931189e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54503415146903e-1

x0 = 3/4*pi, Power = 3/10, lamda = 1, q = 1

$\sin(x)$

n = 10 , left side = 4.06029054897750e-3
1/n^(3/10) = 5.01187233627272e-1
difference = 4.97126943078295e-1
n = 20 , left side = 1.02946339325216e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.06061068143652e-1

```

n = 50 , left side = 1.59207524128235e-4
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.09090287186863e-1
n = 100 , left side = 3.92686042182122e-5
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.51149374546740e-1
n = 200 , left side = 1.13627155432505e-5
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.04017214621294e-1
n = 500 , left side = 1.89942238126850e-6
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54989999332452e-1

```

$\cos(x)$

```

n = 10 , left side = 4.07812534676066e-3
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.97109108280512e-1
n = 20 , left side = 1.01328357734054e-3
          1/n^(3/10) = 4.07090531536904e-1
          difference = 4.06077247959564e-1
n = 50 , left side = 1.67763851029901e-4
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.09081730859962e-1
n = 100 , left side = 4.24032634578353e-5
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.51146239887500e-1
n = 200 , left side = 9.07022584961670e-6
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.04019507110987e-1
n = 500 , left side = 1.37203563288502e-6
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54990526719201e-1

```

$x_0 = 3/4\pi$, Power = 1/2, lamda = 1/4, q = 1/4

$\sin(x)$

n = 10 , left side = 2.00168857534962e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 1.16058908481876e-1
n = 20 , left side = 1.13262072722325e-1
1/n^(1/2) = 2.23606797749979e-1
difference = 1.10344725027654e-1
n = 50 , left side = 4.20732246994345e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 9.93481315378750e-2
n = 100 , left side = 2.03378378667144e-2
1/n^(1/2) = 1.00000000000000e-1
difference = 7.96621621332856e-2
n = 200 , left side = 9.98779497231383e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.07228831463409e-2
n = 500 , left side = 3.95085911348803e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.07705004365078e-2

$\cos(x)$

n = 10 , left side = 9.93951714238188e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.16832594593019e-1
n = 20 , left side = 7.72328322774641e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.46373965472515e-1
n = 50 , left side = 3.60961128316283e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.05325243405681e-1
n = 100 , left side = 1.88410272392823e-2

```

1/n^(1/2) = 1.000000000000000e-1
difference = 8.11589727607177e-2
n = 200 , left side = 9.61343382830904e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.10972442903457e-2
n = 500 , left side = 3.89095422715435e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.08304053228414e-2

```

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1/4, q = 1/2

```

sin(x)
n = 10 , left side = 1.13322309591826e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 2.02905456425012e-1
n = 20 , left side = 6.09253390188926e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.62681458731086e-1
n = 50 , left side = 2.17255059254714e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.19695850311838e-1
n = 100 , left side = 1.03403776602992e-2
1/n^(1/2) = 1.00000000000000e-1
difference = 8.96596223397008e-2
n = 200 , left side = 5.03665536910580e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.56740227495489e-2
n = 500 , left side = 1.98226000171664e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.27390995482792e-2

```

cos(x)

```

n = 10 , left side = 3.12663766609833e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.84961389355855e-1
n = 20 , left side = 3.44627783783421e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.89144019371637e-1
n = 50 , left side = 1.73741901387926e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.24047166098517e-1
n = 100 , left side = 9.25093740382432e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.07490625961757e-2
n = 200 , left side = 4.76419447265553e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.59464836459992e-2
n = 500 , left side = 1.93866173914425e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.27826978108515e-2

```

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1/4, q = 1

$\sin(x)$

```

n = 10 , left side = 2.86823828109783e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.87545383205860e-1
n = 20 , left side = 1.14200334403044e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.12186764309675e-1
n = 50 , left side = 1.90462132680691e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.39516734910503e-1
n = 100 , left side = 4.76820716817317e-4

```

```

1/n^(1/2) = 1.000000000000000e-1
difference = 9.95231792831827e-2
n = 200 , left side = 1.19246801639772e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.05914313170150e-2
n = 500 , left side = 1.90813536952161e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47022781963006e-2

```

$\cos(x)$

```

n = 10 , left side = 5.13589058368008e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.64868860180037e-1
n = 20 , left side = 1.19286950374289e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.11678102712550e-1
n = 50 , left side = 1.90462392585333e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.39516732311456e-1
n = 100 , left side = 4.76820716817428e-4
1/n^(1/2) = 1.000000000000000e-1
difference = 9.95231792831826e-2
n = 200 , left side = 1.19246801639328e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.05914313170154e-2
n = 500 , left side = 1.90813536953272e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47022781963005e-2

```

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1/2, q = 1/4

$\sin(x)$

$n = 10$, left side = 1.14144646548721e-1
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.02083119468117e-1
 $n = 20$, left side = 5.36350821680255e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 1.69971715581953e-1
 $n = 50$, left side = 2.03721764988826e-2
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.21049179738427e-1
 $n = 100$, left side = 9.99650895754645e-3
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.00034910424536e-2
 $n = 200$, left side = 4.95003907205882e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.57606390465959e-2
 $n = 500$, left side = 1.96834136039681e-3
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.27530181895990e-2

$$\cos(x)$$

$n = 10$, left side = 7.63132390190502e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.39914526997788e-1
 $n = 20$, left side = 4.38690929008562e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 1.79737704849123e-1
 $n = 50$, left side = 1.88047305777048e-2
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.22616625659605e-1
 $n = 100$, left side = 9.60447484518001e-3
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.03955251548200e-2
 $n = 200$, left side = 4.85201975765370e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.58586583610010e-2

```

n = 500 , left side = 1.95265778643006e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.27687017635657e-2

```

```

-----
x0 = 3/4*pi, Power = 1/2, lamda = 1/2,  q = 1/2
-----

```

$\sin(x)$

```

n = 10 , left side = 6.18104658618466e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.54417300154991e-1
n = 20 , left side = 2.80061711992697e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.95600626550709e-1
n = 50 , left side = 1.03752156519760e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.31046140585334e-1
n = 100 , left side = 5.04543144574476e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.49545685542552e-2
n = 200 , left side = 2.48679978426469e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.82238783343901e-2
n = 500 , left side = 9.86054094334721e-4
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.37353054556611e-2

```

$\cos(x)$

```

n = 10 , left side = 3.35723331340523e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.82655432882786e-1
n = 20 , left side = 2.07751849096550e-2

```

```

1/n^(1/2) = 2.23606797749979e-1
difference = 2.02831612840324e-1
n = 50 , left side = 9.21511983819845e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.32206236399111e-1
n = 100 , left side = 4.75529632502636e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.52447036749736e-2
n = 200 , left side = 2.41425905034953e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.82964190683052e-2
n = 500 , left side = 9.74447265982392e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.37469122840134e-2

```

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1/2, q = 1

$\sin(x)$

```

n = 10 , left side = 1.23317176677876e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 3.03896048349050e-1
n = 20 , left side = 3.19271958223932e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.20414078167740e-1
n = 50 , left side = 5.12151488168966e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.40909204749141e-1
n = 100 , left side = 1.28083959738734e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.98719160402613e-2
n = 200 , left side = 3.20240359172885e-5
1/n^(1/2) = 7.07106781186548e-2

```

```

difference = 7.06786540827375e-2
n = 500 , left side = 5.12398355012600e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47162355664457e-2

```

$\cos(x)$

```

n = 10 , left side = 1.27760703918800e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 3.03451695624958e-1
n = 20 , left side = 3.19296312784956e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.20413834622129e-1
n = 50 , left side = 5.12152374566921e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.40909203862743e-1
n = 100 , left side = 1.28084284152230e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.98719157158478e-2
n = 200 , left side = 3.20237987473382e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06786543199074e-2
n = 500 , left side = 5.12392899754133e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47162356209983e-2

```

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1, q = 1/4

$\sin(x)$

```

n = 10 , left side = 5.44764920559380e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.61751273960900e-1

```

$n = 20$, left side = $2.59004027714145e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $1.97706394978564e-1$
 $n = 50$, left side = $1.00311377175122e-2$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $1.31390218519797e-1$
 $n = 100$, left side = $4.96093970818390e-3$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $9.50390602918161e-2$
 $n = 200$, left side = $2.46459813024336e-3$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $6.82460799884114e-2$
 $n = 500$, left side = $9.82124425668585e-4$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.37392351243272e-2$

$$\cos(x)$$

$n = 10$, left side = $4.29522822629749e-2$
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = $2.73275483753863e-1$
 $n = 20$, left side = $2.30109143999059e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = $2.00595883350073e-1$
 $n = 50$, left side = $9.56870414660826e-3$
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = $1.31852652090701e-1$
 $n = 100$, left side = $4.84524222640814e-3$
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = $9.51547577735919e-2$
 $n = 200$, left side = $2.43566015728058e-3$
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = $6.82750179613742e-2$
 $n = 500$, left side = $9.77496918584397e-4$
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = $4.37438626314114e-2$

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1, q = 1/2

$\sin(x)$

n = 10 , left side = 2.88447866653312e-2
 1/n^(1/2) = 3.16227766016838e-1
 difference = 2.87382979351507e-1
 n = 20 , left side = 1.33629574745268e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 2.10243840275452e-1
 n = 50 , left side = 5.08446410303787e-3
 1/n^(1/2) = 1.41421356237310e-1
 difference = 1.36336892134272e-1
 n = 100 , left side = 2.49515406149969e-3
 1/n^(1/2) = 1.00000000000000e-1
 difference = 9.75048459385003e-2
 n = 200 , left side = 1.23598531292535e-3
 1/n^(1/2) = 7.07106781186548e-2
 difference = 6.94746928057294e-2
 n = 500 , left side = 4.92089942069840e-4
 1/n^(1/2) = 4.47213595499958e-2
 difference = 4.42292696079260e-2

$\cos(x)$

n = 10 , left side = 1.98502218336606e-2
 1/n^(1/2) = 3.16227766016838e-1
 difference = 2.96377544183177e-1
 n = 20 , left side = 1.11111768757201e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 2.12495620874259e-1
 n = 50 , left side = 4.72342001665071e-3

```

1/n^(1/2) = 1.41421356237310e-1
difference = 1.36697936220659e-1
n = 100 , left side = 2.40486942054952e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.75951305794505e-2
n = 200 , left side = 1.21344788187139e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.94972302367834e-2
n = 500 , left side = 4.88483607931189e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.42328759420646e-2

```

 $x_0 = 3/4\pi$, Power = 1/2, lamda = 1, q = 1

$\sin(x)$

```

n = 10 , left side = 4.06029054897750e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.12167475467860e-1
n = 20 , left side = 1.02946339325216e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.22577334356727e-1
n = 50 , left side = 1.59207524128235e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41262148713181e-1
n = 100 , left side = 3.92686042182122e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99607313957818e-2
n = 200 , left side = 1.13627155432505e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06993154031115e-2
n = 500 , left side = 1.89942238126850e-6
1/n^(1/2) = 4.47213595499958e-2

```

difference = 4.47194601276145e-2

$\cos(x)$

n = 10 , left side = 4.07812534676066e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.12149640670077e-1
n = 20 , left side = 1.01328357734054e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.22593514172638e-1
n = 50 , left side = 1.67763851029901e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41253592386280e-1
n = 100 , left side = 4.24032634578353e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99575967365422e-2
n = 200 , left side = 9.07022584961670e-6
1/n^(1/2) = 7.07106781186548e-2
difference = 7.07016078928051e-2
n = 500 , left side = 1.37203563288502e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47199875143629e-2

x0 = 3/4*pi, Power = 7/10, lamda = 1/4, q = 1/4

$\sin(x)$

n = 10 , left side = 2.00168857534962e-1
1/n^(7/10) = 1.99526231496888e-1
difference = -6.42626038073607e-4
n = 20 , left side = 1.13262072722325e-1
1/n^(7/10) = 1.22822802611579e-1
difference = 9.56072988925424e-3


```

n = 50 , left side = 4.20732246994345e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 2.25994759583012e-2
n = 100 , left side = 2.03378378667144e-2
          1/n^(7/10) = 3.98107170553497e-2
          difference = 1.94728791886353e-2
n = 200 , left side = 9.98779497231383e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 1.45185759746607e-2
n = 500 , left side = 3.95085911348803e-3
          1/n^(7/10) = 1.29039002429643e-2
          difference = 8.95304112947630e-3

```

$\cos(x)$

```

n = 10 , left side = 9.93951714238188e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.00131060073069e-1
n = 20 , left side = 7.72328322774641e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 4.55899703341149e-2
n = 50 , left side = 3.60961128316283e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 2.85765878261075e-2
n = 100 , left side = 1.88410272392823e-2
          1/n^(7/10) = 3.98107170553497e-2
          difference = 2.09696898160675e-2
n = 200 , left side = 9.61343382830904e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 1.48929371186655e-2
n = 500 , left side = 3.89095422715435e-3
          1/n^(7/10) = 1.29039002429643e-2
          difference = 9.01294601580997e-3

```

$x_0 = 3/4\pi$, Power = 7/10, lamda = 1/4, q = 1/2

$\sin(x)$

```
n = 10 , left side = 1.13322309591826e-1
          1/n^(7/10) = 1.99526231496888e-1
          difference = 8.62039219050625e-2
n = 20 , left side = 6.09253390188926e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 6.18974635926864e-2
n = 50 , left side = 2.17255059254714e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 4.29471947322644e-2
n = 100 , left side = 1.03403776602992e-2
          1/n^(7/10) = 3.98107170553497e-2
          difference = 2.94703393950506e-2
n = 200 , left side = 5.03665536910580e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 1.94697155778687e-2
n = 500 , left side = 1.98226000171664e-3
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.09216402412477e-2
```

$\cos(x)$

```
n = 10 , left side = 3.12663766609833e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.68259854835905e-1
n = 20 , left side = 3.44627783783421e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 8.83600242332369e-2
n = 50 , left side = 1.73741901387926e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 4.72985105189432e-2
n = 100 , left side = 9.25093740382432e-3
```

```

1/n^(7/10) = 3.98107170553497e-2
difference = 3.05597796515254e-2
n = 200 , left side = 4.76419447265553e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.97421764743190e-2
n = 500 , left side = 1.93866173914425e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.09652385038201e-2

```

```

x0 = 3/4*pi, Power = 7/10, lamda = 1/4, q = 1

```

$\sin(x)$

```

n = 10 , left side = 2.86823828109783e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.70843848685910e-1
n = 20 , left side = 1.14200334403044e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.11402769171275e-1
n = 50 , left side = 1.90462132680691e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 6.27680793309288e-2
n = 100 , left side = 4.76820716817317e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.93338963385324e-2
n = 200 , left side = 1.19246801639772e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.43871241453347e-2
n = 500 , left side = 1.90813536952161e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28848188892691e-2

```

$\cos(x)$

```

n = 10 , left side = 5.13589058368008e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.48167325660087e-1
n = 20 , left side = 1.19286950374289e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.10894107574150e-1
n = 50 , left side = 1.90462392585333e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.27680767318824e-2
n = 100 , left side = 4.76820716817428e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.93338963385323e-2
n = 200 , left side = 1.19246801639328e-4
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.43871241453352e-2
n = 500 , left side = 1.90813536953272e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28848188892690e-2

```

 $x_0 = 3/4\pi$, Power = 7/10, lamda = 1/2, q = 1/4

$\sin(x)$

```

n = 10 , left side = 1.14144646548721e-1
          1/n^(7/10) = 1.99526231496888e-1
          difference = 8.53815849481666e-2
n = 20 , left side = 5.36350821680255e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 6.91877204435536e-2
n = 50 , left side = 2.03721764988826e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 4.43005241588532e-2
n = 100 , left side = 9.99650895754645e-3

```

```

1/n^(7/10) = 3.98107170553497e-2
difference = 2.98142080978033e-2
n = 200 , left side = 4.95003907205882e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.95563318749157e-2
n = 500 , left side = 1.96834136039681e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.09355588825675e-2

```

$\cos(x)$

```

n = 10 , left side = 7.63132390190502e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.23212992477838e-1
n = 20 , left side = 4.38690929008562e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 7.89537097107229e-2
n = 50 , left side = 1.88047305777048e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 4.58679700800309e-2
n = 100 , left side = 9.60447484518001e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.02062422101697e-2
n = 200 , left side = 4.85201975765370e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.96543511893208e-2
n = 500 , left side = 1.95265778643006e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.09512424565343e-2

```

 $x_0 = 3/4\pi$, Power = 7/10, lamda = 1/2, q = 1/2

$\sin(x)$

$n = 10$, left side = $6.18104658618466e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.37715765635041e-1$
 $n = 20$, left side = $2.80061711992697e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $9.48166314123093e-2$
 $n = 50$, left side = $1.03752156519760e-2$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $5.42974850057598e-2$
 $n = 100$, left side = $5.04543144574476e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.47652856096050e-2$
 $n = 200$, left side = $2.48679978426469e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.20195711627098e-2$
 $n = 500$, left side = $9.86054094334721e-4$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.19178461486296e-2$

$\cos(x)$

$n = 10$, left side = $3.35723331340523e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.65953898362836e-1$
 $n = 20$, left side = $2.07751849096550e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $1.02047617701924e-1$
 $n = 50$, left side = $9.21511983819845e-3$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $5.54575808195373e-2$
 $n = 100$, left side = $4.75529632502636e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.50554207303234e-2$
 $n = 200$, left side = $2.41425905034953e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.20921118966250e-2$

```

n = 500 , left side = 9.74447265982392e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.19294529769819e-2

```

```

-----
x0 = 3/4*pi, Power = 7/10, lamda = 1/2,  q = 1
-----

```

$\sin(x)$

```

n = 10 , left side = 1.23317176677876e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.87194513829100e-1
n = 20 , left side = 3.19271958223932e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.19630083029340e-1
n = 50 , left side = 5.12151488168966e-4
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.41605491695668e-2
n = 100 , left side = 1.28083959738734e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.96826330956110e-2
n = 200 , left side = 3.20240359172885e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44743469110572e-2
n = 500 , left side = 5.12398355012600e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28987762594142e-2

```

$\cos(x)$

```

n = 10 , left side = 1.27760703918800e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.86750161105008e-1
n = 20 , left side = 3.19296312784956e-3

```

```

1/n^(7/10) = 1.22822802611579e-1
difference = 1.19629839483729e-1
n = 50 , left side = 5.12152374566921e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.41605482831688e-2
n = 100 , left side = 1.28084284152230e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.96826327711975e-2
n = 200 , left side = 3.20237987473382e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44743471482272e-2
n = 500 , left side = 5.12392899754133e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28987763139668e-2

```

 $x_0 = 3/4\pi$, Power = 7/10, lamda = 1, q = 1/4

```

sin(x)

n = 10 , left side = 5.44764920559380e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.45049739440950e-1
n = 20 , left side = 2.59004027714145e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 9.69223998401646e-2
n = 50 , left side = 1.00311377175122e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 5.46415629402236e-2
n = 100 , left side = 4.96093970818390e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.48497773471658e-2
n = 200 , left side = 2.46459813024336e-3
1/n^(7/10) = 2.45063709469745e-2

```



```

        difference = 2.20417728167311e-2
n = 500 , left side = 9.82124425668585e-4
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.19217758172957e-2

```

$\cos(x)$

```

n = 10 , left side = 4.29522822629749e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.56573949233913e-1
n = 20 , left side = 2.30109143999059e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 9.98118882116732e-2
n = 50 , left side = 9.56870414660826e-3
        1/n^(7/10) = 6.46727006577358e-2
        difference = 5.51039965111275e-2
n = 100 , left side = 4.84524222640814e-3
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.49654748289416e-2
n = 200 , left side = 2.43566015728058e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.20707107896939e-2
n = 500 , left side = 9.77496918584397e-4
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.19264033243799e-2

```

```

x0 = 3/4*pi, Power = 7/10, lamda = 1,  q = 1/2

```

$\sin(x)$

```

n = 10 , left side = 2.88447866653312e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.70681444831557e-1

```

$n = 20$, left side = $1.33629574745268e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $1.09459845137052e-1$
 $n = 50$, left side = $5.08446410303787e-3$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $5.95882365546979e-2$
 $n = 100$, left side = $2.49515406149969e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.73155629938500e-2$
 $n = 200$, left side = $1.23598531292535e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.32703856340492e-2$
 $n = 500$, left side = $4.92089942069840e-4$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.24118103008945e-2$

$$\cos(x)$$

$n = 10$, left side = $1.98502218336606e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.79676009663227e-1$
 $n = 20$, left side = $1.11111768757201e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $1.11711625735859e-1$
 $n = 50$, left side = $4.72342001665071e-3$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $5.99492806410850e-2$
 $n = 100$, left side = $2.40486942054952e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.74058476348002e-2$
 $n = 200$, left side = $1.21344788187139e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.32929230651031e-2$
 $n = 500$, left side = $4.88483607931189e-4$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.24154166350331e-2$

 $x_0 = 3/4\pi$, Power = 7/10, lamda = 1, q = 1

$\sin(x)$

```

n = 10 , left side = 4.06029054897750e-3
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.95465940947911e-1
n = 20 , left side = 1.02946339325216e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.21793339218327e-1
n = 50 , left side = 1.59207524128235e-4
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.45134931336075e-2
n = 100 , left side = 3.92686042182122e-5
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.97714484511315e-2
n = 200 , left side = 1.13627155432505e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44950082314313e-2
n = 500 , left side = 1.89942238126850e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.29020008205831e-2

```

$\cos(x)$

```

n = 10 , left side = 4.07812534676066e-3
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.95448106150127e-1
n = 20 , left side = 1.01328357734054e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.21809519034239e-1
n = 50 , left side = 1.67763851029901e-4

```

```

1/n^(7/10) = 6.46727006577358e-2
difference = 6.45049368067059e-2
n = 100 , left side = 4.24032634578353e-5
1/n^(7/10) = 3.98107170553497e-2
difference = 3.97683137918919e-2
n = 200 , left side = 9.07022584961670e-6
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44973007211249e-2
n = 500 , left side = 1.37203563288502e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29025282073314e-2

```

```

[ ]: RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1]    #deformation parameter lamda over (0,
    ↪ 1] - these are the beta values in the formula
qs = [1/4, 1/2, 1]    #deformation coefficient

funcs = [x^(1/3), x, x^2, x^3, x^4, x^10]    #choice of
    ↪ functions
a = -1    #the interval
b = 1    #the interval
x0=1/2

for power in powers:
    ↪
    ↪#####
    for lamda in lamdas:    #going over each lamda value
        ↪
        ↪#####
        for q in qs:        #going over each q value

```

```

    ↪ #####
        print()
        print()
    ↪
    ↪ print("-----
        print("x0 = " + str(x0)+", Power = "+ str(power)+
    ↪ ", lamda = "+ str(lamda) + ", q = " + str(q))
    ↪
    ↪ print("-----

        #the activation function
        phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
    ↪ (e^(lamda*x)+q*e^(-lamda*x))    #formula 18.1

        #
        G(x) = 1/4*(phi(x+1) - phi(x-1))    #formula 18.9

    ↪ #####
        for i in range(len(funcs)):
    ↪ #####
            f(x)=funcs[i]
            show(f(x))
            for n in [10, 20, 50, 100, 200, 500]:
                #def L(n, f, x):    #real-valued linear
    ↪ neural network operators
                    #    return sum(f(k/n)*G(n*x-k) for k in
    ↪ [ceil(n*a),...,floor(n*b)]) / sum(G(n*x-k) for k in
    ↪ [ceil(n*a),...,floor(n*b)])
                    #leftSide = abs(L(n,f,x0)-f(x0))

```

```

        leftSide = abs(sum(f(k/n)*G(n*x0-k) for k
↪in [ceil(n*a),...,floor(n*b)])/sum(G(n*x0-k) for k in
↪[ceil(n*a),...,floor(n*b)]))-f(x0))
        val1 = n
        val2 = leftSide.n()
        val3 = 1/(n^power).n()
        print("          n = "+str(val1), ", left
↪side = "+str(val2), "\n          1/
↪n^("+str(power)+") = "+str(val3), "\n
↪difference = "+str(val3-val2))

```

x0 = 1/2, Power = 3/10, lamda = 1/4, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 4.65939350634126e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.54593298563860e-1
n = 20 , left side = 5.39175273339064e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.53173004202998e-1
n = 50 , left side = 2.66706958509516e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.82578798860040e-1
n = 100 , left side = 1.39692422376505e-2
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.37219400913308e-1
n = 200 , left side = 7.15466879542836e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.96873908541409e-1
n = 500 , left side = 2.90466167844949e-3
          1/n^(3/10) = 1.54991898754834e-1

```

difference = 1.52087237076384e-1

x

n = 10 , left side = 1.46960541508878e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.54226692118394e-1
n = 20 , left side = 1.28044818944893e-1
1/n^(3/10) = 4.07090531536904e-1
difference = 2.79045712592011e-1
n = 50 , left side = 5.54458428059442e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.53803651905048e-1
n = 100 , left side = 2.77258872001896e-2
1/n^(3/10) = 2.51188643150958e-1
difference = 2.23462755950768e-1
n = 200 , left side = 1.38629436111991e-2
1/n^(3/10) = 2.04028577336837e-1
difference = 1.90165633725638e-1
n = 500 , left side = 5.54517744447935e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.49446721310354e-1

x^2

n = 10 , left side = 2.44068957215094e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 2.57118276412179e-1
n = 20 , left side = 1.73393496362237e-1
1/n^(3/10) = 4.07090531536904e-1
difference = 2.33697035174667e-1
n = 50 , left side = 6.39143140576404e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.45335180653351e-1
n = 100 , left side = 2.98438925970291e-2
1/n^(3/10) = 2.51188643150958e-1

```

        difference = 2.21344750553929e-1
n = 200 , left side = 1.43924449634693e-2
        1/n^(3/10) = 2.04028577336837e-1
        difference = 1.89636132373368e-1
n = 500 , left side = 5.62989766084276e-3
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.49362001093991e-1

```

$$x^3$$

```

n = 10 , left side = 2.70532081486956e-1
        1/n^(3/10) = 5.01187233627272e-1
        difference = 2.30655152140317e-1
n = 20 , left side = 1.75303858260733e-1
        1/n^(3/10) = 4.07090531536904e-1
        difference = 2.31786673276171e-1
n = 50 , left side = 5.53534166881951e-2
        1/n^(3/10) = 3.09249494710992e-1
        difference = 2.53896078022797e-1
n = 100 , left side = 2.41049670707777e-2
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.27083676080180e-1
n = 200 , left side = 1.12081526845386e-2
        1/n^(3/10) = 2.04028577336837e-1
        difference = 1.92820424652298e-1
n = 500 , left side = 4.28703175655951e-3
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.50704866998274e-1

```

$$x^4$$

```

n = 10 , left side = 2.71811655321145e-1
        1/n^(3/10) = 5.01187233627272e-1
        difference = 2.29375578306128e-1
n = 20 , left side = 1.59700780897926e-1
        1/n^(3/10) = 4.07090531536904e-1

```



```

        difference = 2.47389750638979e-1
n = 50 , left side = 4.27867893894554e-2
        1/n^(3/10) = 3.09249494710992e-1
        difference = 2.66462705321536e-1
n = 100 , left side = 1.73213915349858e-2
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.33867251615972e-1
n = 200 , left side = 7.76000677274204e-3
        1/n^(3/10) = 2.04028577336837e-1
        difference = 1.96268570564095e-1
n = 500 , left side = 2.90182870840223e-3
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.52090070046431e-1

```

$$x^{10}$$

```

n = 10 , left side = 1.86850823331552e-1
        1/n^(3/10) = 5.01187233627272e-1
        difference = 3.14336410295721e-1
n = 20 , left side = 6.58347789205075e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.41255752616397e-1
n = 50 , left side = 4.87085072187464e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.04378643989117e-1
n = 100 , left side = 1.09970741430996e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.50088935736648e-1
n = 200 , left side = 3.82804893814586e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03645772443022e-1
n = 500 , left side = 1.24276968148264e-4
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54867621786685e-1

```

x0 = 1/2, Power = 3/10, lamda = 1/4, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.96080052578450e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.81579228369427e-1
n = 20 , left side = 2.08299179386359e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.86260613598269e-1
n = 50 , left side = 1.26071386109592e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.96642356100033e-1
n = 100 , left side = 6.81015763376169e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.44378485517196e-1
n = 200 , left side = 3.53432821873112e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.00494249118106e-1
n = 500 , left side = 1.44549650366488e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.53546402251169e-1

```

$$x$$

```

n = 10 , left side = 5.68097172543791e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.44377516372893e-1
n = 20 , left side = 6.32752819000137e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.43815249636891e-1
n = 50 , left side = 2.77227554082915e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.81526739302700e-1

```

$n = 100$, left side = 1.38629435997853e-2
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = 2.37325699551173e-1
 $n = 200$, left side = 6.93147180559972e-3
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = 1.97097105531237e-1
 $n = 500$, left side = 2.77258872223995e-3
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = 1.52219310032594e-1

$$x^2$$

$n = 10$, left side = 1.47872752410878e-1
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = 3.53314481216394e-1
 $n = 20$, left side = 9.78605569036922e-2
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = 3.09229974633212e-1
 $n = 50$, left side = 3.38868135786705e-2
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = 2.75362681132321e-1
 $n = 100$, left side = 1.54044053860309e-2
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = 2.35784237764927e-1
 $n = 200$, left side = 7.31683725369431e-3
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = 1.96711740083143e-1
 $n = 500$, left side = 2.83424719393494e-3
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = 1.52157651560899e-1

$$x^3$$

$n = 10$, left side = 1.73192000431426e-1
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = 3.27995233195847e-1

```

n = 20 , left side = 1.03864925521532e-1
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.03225606015372e-1
n = 50 , left side = 3.05073733511566e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.78742121359835e-1
n = 100 , left side = 1.27681795752464e-2
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.38420463575712e-1
n = 200 , left side = 5.78399942625288e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.98244577910584e-1
n = 500 , left side = 2.17239948281706e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.52819499272017e-1

```

$$x^4$$

```

n = 10 , left side = 1.80257354696213e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.20929878931060e-1
n = 20 , left side = 9.63498090064417e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.10740722530463e-1
n = 50 , left side = 2.41914336230855e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.85058061087906e-1
n = 100 , left side = 9.37035404303910e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.41818289107919e-1
n = 200 , left side = 4.05904957310241e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.99969527763735e-1
n = 500 , left side = 1.47973714572865e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.53512161609105e-1

```

$$x^{10}$$

```

n = 10 , left side = 1.19260700953333e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.81926532673939e-1
n = 20 , left side = 3.73465208237525e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.69744010713152e-1
n = 50 , left side = 2.80162956157206e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.06447865149420e-1
n = 100 , left side = 6.32905221145845e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50555737929812e-1
n = 200 , left side = 2.12043953336508e-4
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03816533383500e-1
n = 500 , left side = 6.54808182457762e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54926417936588e-1

```

x0 = 1/2, Power = 3/10, lamda = 1/4, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 8.10134985303516e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.20173735096921e-1
n = 20 , left side = 1.68560763315217e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.90234455205383e-1
n = 50 , left side = 1.97802106438832e-3
          1/n^(3/10) = 3.09249494710992e-1

```

```

        difference = 3.07271473646603e-1
n = 100 , left side = 4.80026592954430e-4
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.50708616558004e-1
n = 200 , left side = 1.19239646471425e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03909337690366e-1
n = 500 , left side = 1.90449720897101e-5
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54972853782744e-1

```

x

```

n = 10 , left side = 4.91637605442458e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.52023473083027e-1
n = 20 , left side = 3.39394250680042e-3
        1/n^(3/10) = 4.07090531536904e-1
        difference = 4.03696589030104e-1
n = 50 , left side = 1.64890433890230e-6
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.09247845806653e-1
n = 100 , left side = 5.86114490275236e-12
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.51188643145097e-1
n = 200 , left side = 0.000000000000000e0
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.04028577336837e-1
n = 500 , left side = 0.000000000000000e0
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54991898754834e-1

```

x^2

```

n = 10 , left side = 5.25230782287648e-2
        1/n^(3/10) = 5.01187233627272e-1

```

```

        difference = 4.48664155398508e-1
n = 20 , left side = 2.83373415030339e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.78753190033871e-1
n = 50 , left side = 5.39457664221732e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.03854918068774e-1
n = 100 , left side = 1.34928057788158e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.49839362573076e-1
n = 200 , left side = 3.37320146703013e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03691257190134e-1
n = 500 , left side = 5.39712234724488e-5
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54937927531361e-1

```

$$x^3$$

```

n = 10 , left side = 8.22234506283697e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.18963782998903e-1
n = 20 , left side = 4.36057626242193e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.63484768912685e-1
n = 50 , left side = 8.09259320656272e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.01156901504429e-1
n = 100 , left side = 2.02392086959333e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.49164722281365e-1
n = 200 , left side = 5.05980220054492e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03522597116782e-1
n = 500 , left side = 8.09568352087009e-5
        1/n^(3/10) = 1.54991898754834e-1

```

difference = 1.54910941919625e-1

$$x^4$$

n = 10 , left side = 1.01052973293562e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 4.00134260333710e-1
n = 20 , left side = 4.67481413489399e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.60342390187965e-1
n = 50 , left side = 8.21282675171919e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.01036667959273e-1
n = 100 , left side = 2.03145927015694e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.49157183880801e-1
n = 200 , left side = 5.06451370153385e-4
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03522125966684e-1
n = 500 , left side = 8.09688966512678e-5
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54910929858182e-1

$$x^{10}$$

n = 10 , left side = 7.01207124827065e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.31066521144566e-1
n = 20 , left side = 2.01574704246336e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.86933061112271e-1
n = 50 , left side = 1.42480124701184e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.07824693463980e-1
n = 100 , left side = 2.63178146244861e-4
1/n^(3/10) = 2.51188643150958e-1


```

difference = 2.50925465004713e-1
n = 200 , left side = 6.08599933529959e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03967717343484e-1
n = 500 , left side = 9.52678518000342e-6
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54982371969654e-1

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 5.33369660315088e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.47850267595764e-1
n = 20 , left side = 3.23728371506258e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.74717694386279e-1
n = 50 , left side = 1.39366391997124e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.95312855511279e-1
n = 100 , left side = 7.14621856758857e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.44042424583369e-1
n = 200 , left side = 3.61961230894126e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.00408965027896e-1
n = 500 , left side = 1.45930076253686e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53532597992297e-1

```

$$x$$

```

n = 10 , left side = 1.29071817455777e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.72115416171495e-1
n = 20 , left side = 6.92514035886481e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.37839127948256e-1
n = 50 , left side = 2.77258876437841e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.81523607067208e-1
n = 100 , left side = 1.38629438314646e-2
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.37325699319493e-1
n = 200 , left side = 6.93147191573251e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.97097105421104e-1
n = 500 , left side = 2.77258876629327e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.52219309988540e-1

```

$$x^2$$

```

n = 10 , left side = 1.77390913518807e-1
          1/n^(3/10) = 5.01187233627272e-1
          difference = 3.23796320108465e-1
n = 20 , left side = 8.30737213816240e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.24016810155280e-1
n = 50 , left side = 2.99438930027937e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.79305601708198e-1
n = 100 , left side = 1.44174451738843e-2
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.36771197977074e-1
n = 200 , left side = 7.07009725133750e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 1.96958480085499e-1

```

n = 500 , left side = 2.79476881998975e-3
 1/n^(3/10) = 1.54991898754834e-1
 difference = 1.52197129934844e-1

$$x^3$$

n = 10 , left side = 1.81706840017598e-1
 1/n^(3/10) = 5.01187233627272e-1
 difference = 3.19480393609674e-1
 n = 20 , left side = 7.48642399521749e-2
 1/n^(3/10) = 4.07090531536904e-1
 difference = 3.32226291584729e-1
 n = 50 , left side = 2.42632851043290e-2
 1/n^(3/10) = 3.09249494710992e-1
 difference = 2.84986209606663e-1
 n = 100 , left side = 1.12466925545658e-2
 1/n^(3/10) = 2.51188643150958e-1
 difference = 2.39941950596392e-1
 n = 200 , left side = 5.40875852362388e-3
 1/n^(3/10) = 2.04028577336837e-1
 difference = 1.98619818813213e-1
 n = 500 , left side = 2.11285351660367e-3
 1/n^(3/10) = 1.54991898754834e-1
 difference = 1.52879045238230e-1

$$x^4$$

n = 10 , left side = 1.67787178069694e-1
 1/n^(3/10) = 5.01187233627272e-1
 difference = 3.33400055557578e-1
 n = 20 , left side = 6.03397370785075e-2
 1/n^(3/10) = 4.07090531536904e-1
 difference = 3.46750794458397e-1
 n = 50 , left side = 1.74893080184918e-2
 1/n^(3/10) = 3.09249494710992e-1
 difference = 2.91760186692500e-1

```

n = 100 , left side = 7.79966635748697e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.43388976793471e-1
n = 200 , left side = 3.67816819644864e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.00350409140388e-1
n = 500 , left side = 1.41984974971847e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.53572049005115e-1

```

$$x^{10}$$

```

n = 10 , left side = 7.68027797084298e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.24384453918843e-1
n = 20 , left side = 9.76146773173466e-3
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.97329063805170e-1
n = 50 , left side = 1.13068528957820e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.08118809421414e-1
n = 100 , left side = 3.88474877995726e-4
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.50800168272962e-1
n = 200 , left side = 1.62038602246220e-4
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.03866538734591e-1
n = 500 , left side = 5.81892063837631e-5
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54933709548450e-1

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.96445247579275e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.81542708869345e-1
n = 20 , left side = 1.48888789279412e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.92201652608963e-1
n = 50 , left side = 6.77605354912070e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.02473441161871e-1
n = 100 , left side = 3.52568356510685e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.47662959585851e-1
n = 200 , left side = 1.79801315131500e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.02230564185522e-1
n = 500 , left side = 7.27768804897111e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54264129949937e-1

```

$$x$$

```

n = 10 , left side = 6.38935662496244e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.37293667377648e-1
n = 20 , left side = 3.46235470985199e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.72466984438385e-1
n = 50 , left side = 1.38629429714340e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.95386551739558e-1
n = 100 , left side = 6.93147149063511e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.44257171660323e-1
n = 200 , left side = 3.46573574531772e-3

```

```

1/n^(3/10) = 2.04028577336837e-1
difference = 2.00562841591519e-1
n = 500 , left side = 1.38629429812720e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53605604456707e-1

```

$$x^2$$

```

n = 10 , left side = 1.01231195168683e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.99956038458590e-1
n = 20 , left side = 4.48623512045705e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.62228180332334e-1
n = 50 , left side = 1.55044047114581e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.93745089999534e-1
n = 100 , left side = 7.34183692697737e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.43846806223981e-1
n = 200 , left side = 3.56832710440319e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.00460250232434e-1
n = 500 , left side = 1.40270891558075e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53589189839253e-1

```

$$x^3$$

```

n = 10 , left side = 1.09051180141690e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.92136053485583e-1
n = 20 , left side = 4.22970110547811e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.64793520482123e-1
n = 50 , left side = 1.29223379237350e-2

```

```

1/n^(3/10) = 3.09249494710992e-1
difference = 2.96327156787257e-1
n = 100 , left side = 5.82201903348833e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.45366624117470e-1
n = 200 , left side = 2.75417225524147e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.01274405081595e-1
n = 500 , left side = 1.06440558786389e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53927493166970e-1

```

$$x^4$$

```

n = 10 , left side = 1.02540884681400e-1
1/n^(3/10) = 5.01187233627272e-1
difference = 3.98646348945873e-1
n = 20 , left side = 3.49981874860216e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.72092344050883e-1
n = 50 , left side = 9.52960631552958e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 2.99719888395462e-1
n = 100 , left side = 4.09764755001929e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.47090995600939e-1
n = 200 , left side = 1.88876104704880e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.02139816289788e-1
n = 500 , left side = 7.17895958025183e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54274002796809e-1

```

$$x^{10}$$

```

n = 10 , left side = 4.39432749437958e-2

```

```

1/n^(3/10) = 5.01187233627272e-1
difference = 4.57243958683477e-1
n = 20 , left side = 5.59881198565948e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.01491719551245e-1
n = 50 , left side = 6.58122885628435e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.08591371825363e-1
n = 100 , left side = 2.17135216419696e-4
1/n^(3/10) = 2.51188643150958e-1
difference = 2.50971507934538e-1
n = 200 , left side = 8.67799400496487e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03941797396787e-1
n = 500 , left side = 3.00238003038830e-5
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54961874954530e-1

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.88583358245119e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.82328897802761e-1
n = 20 , left side = 3.43036193768762e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.03660169599217e-1
n = 50 , left side = 5.15745217625541e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.08733749493366e-1
n = 100 , left side = 1.28085535782807e-4
1/n^(3/10) = 2.51188643150958e-1

```



```

        difference = 2.51060557615175e-1
n = 200 , left side = 3.19697272262998e-5
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03996607609611e-1
n = 500 , left side = 5.11285838666886e-6
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54986785896447e-1

```

x

```

n = 10 , left side = 3.03097878388953e-3
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.98156254843383e-1
n = 20 , left side = 1.79813965206499e-5
        1/n^(3/10) = 4.07090531536904e-1
        difference = 4.07072550140384e-1
n = 50 , left side = 5.04973840520506e-12
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.09249494705942e-1
n = 100 , left side = 1.11022302462516e-16
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.51188643150958e-1
n = 200 , left side = 0.0000000000000000e0
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.04028577336837e-1
n = 500 , left side = 5.55111512312578e-17
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54991898754834e-1

```

x^2

```

n = 10 , left side = 3.13288100022530e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.69858423625019e-1
n = 20 , left side = 9.02981956494553e-3
        1/n^(3/10) = 4.07090531536904e-1

```

```

        difference = 3.98060711971959e-1
n = 50 , left side = 1.44928066358080e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.07800214047411e-1
n = 100 , left side = 3.62320167826280e-4
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.50826322983132e-1
n = 200 , left side = 9.05800419565561e-5
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03937997294880e-1
n = 500 , left side = 1.44928067130623e-5
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54977405948121e-1

```

$$x^3$$

```

n = 10 , left side = 4.78600213916086e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.53327212235664e-1
n = 20 , left side = 1.35520533206201e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.93538478216284e-1
n = 50 , left side = 2.17392099772531e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.07075573713266e-1
n = 100 , left side = 5.43480251739309e-4
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.50645162899219e-1
n = 200 , left side = 1.35870062934806e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03892707273902e-1
n = 500 , left side = 2.17392100695102e-5
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54970159544764e-1

```

$$x^4$$

$n = 10$, left side = $5.14533906505907e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.49733842976682e-1$
 $n = 20$, left side = $1.38732244570932e-2$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $3.93217307079811e-1$
 $n = 50$, left side = $2.18227896612364e-3$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $3.07067215744868e-1$
 $n = 100$, left side = $5.44002624822695e-4$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.50644640526135e-1$
 $n = 200$, left side = $1.35902711252578e-4$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.03892674625584e-1$
 $n = 500$, left side = $2.17400458664835e-5$
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = $1.54970158708967e-1$

$$x^{10}$$

$n = 10$, left side = $2.39574413183785e-2$
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = $4.77229792308894e-1$
 $n = 20$, left side = $2.97086004062642e-3$
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = $4.04119671496278e-1$
 $n = 50$, left side = $2.83599622557975e-4$
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = $3.08965895088434e-1$
 $n = 100$, left side = $6.54249762594573e-5$
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = $2.51123218174699e-1$
 $n = 200$, left side = $1.60297404174447e-5$
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = $2.04012547596420e-1$

```

n = 500 , left side = 2.55030753047421e-6
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54989348447303e-1

```

x0 = 1/2, Power = 3/10, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 3.15884348850566e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.69598798742216e-1
n = 20 , left side = 1.69988381319672e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.90091693404937e-1
n = 50 , left side = 7.10929624048973e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.02140198470502e-1
n = 100 , left side = 3.60940309324842e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.47579240057710e-1
n = 200 , left side = 1.81874048780928e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.02209836849028e-1
n = 500 , left side = 7.30913664388266e-4
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.54260985090445e-1

```

$$x$$

```

n = 10 , left side = 6.92279510900182e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.31959282537254e-1
n = 20 , left side = 3.46421335720477e-2

```

```

1/n^(3/10) = 4.07090531536904e-1
difference = 3.72448397964857e-1
n = 50 , left side = 1.38568544082612e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.95392640302730e-1
n = 100 , left side = 6.92842720413067e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.44260215946827e-1
n = 200 , left side = 3.46421360206539e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.00564363734772e-1
n = 500 , left side = 1.38568544082618e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53606213314008e-1

```

$$x^2$$

```

n = 10 , left side = 8.55451218222509e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.15642111805021e-1
n = 20 , left side = 3.87308178004870e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.68359713736417e-1
n = 50 , left side = 1.45110441151024e-2
1/n^(3/10) = 3.09249494710992e-1
difference = 2.94738450595889e-1
n = 100 , left side = 7.09197463084094e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.44096668520117e-1
n = 200 , left side = 3.50510045874292e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.00523476878094e-1
n = 500 , left side = 1.39222733789462e-3
1/n^(3/10) = 1.54991898754834e-1
difference = 1.53599671416939e-1

```

$$x^3$$

```

n = 10 , left side = 7.91097899306038e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.22077443696669e-1
n = 20 , left side = 3.24567771921774e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.74633754344727e-1
n = 50 , left side = 1.13958230614945e-2
          1/n^(3/10) = 3.09249494710992e-1
          difference = 2.97853671649497e-1
n = 100 , left side = 5.44437875504283e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.45744264395915e-1
n = 200 , left side = 2.65983263805031e-3
          1/n^(3/10) = 2.04028577336837e-1
          difference = 2.01368744698787e-1
n = 500 , left side = 1.04909882391729e-3
          1/n^(3/10) = 1.54991898754834e-1
          difference = 1.53942799930916e-1

```

$$x^4$$

```

n = 10 , left side = 6.53260321168637e-2
          1/n^(3/10) = 5.01187233627272e-1
          difference = 4.35861201510409e-1
n = 20 , left side = 2.41899798353885e-2
          1/n^(3/10) = 4.07090531536904e-1
          difference = 3.82900551701516e-1
n = 50 , left side = 7.95482775153313e-3
          1/n^(3/10) = 3.09249494710992e-1
          difference = 3.01294666959459e-1
n = 100 , left side = 3.71509170320668e-3
          1/n^(3/10) = 2.51188643150958e-1
          difference = 2.47473551447751e-1
n = 200 , left side = 1.79412654760111e-3

```

```

1/n^(3/10) = 2.04028577336837e-1
difference = 2.02234450789236e-1
n = 500 , left side = 7.02699493465395e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54289199261368e-1

```

$$x^{10}$$

```

n = 10 , left side = 1.23005431860469e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.88886690441225e-1
n = 20 , left side = 1.95443229260425e-3
1/n^(3/10) = 4.07090531536904e-1
difference = 4.05136099244300e-1
n = 50 , left side = 4.11078488591595e-4
1/n^(3/10) = 3.09249494710992e-1
difference = 3.08838416222400e-1
n = 100 , left side = 1.66923378278077e-4
1/n^(3/10) = 2.51188643150958e-1
difference = 2.51021719772680e-1
n = 200 , left side = 7.51857729315232e-5
1/n^(3/10) = 2.04028577336837e-1
difference = 2.03953391563905e-1
n = 500 , left side = 2.82350791906388e-5
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54963663675643e-1

```

x0 = 1/2, Power = 3/10, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.40292664147630e-2
1/n^(3/10) = 5.01187233627272e-1

```

```

        difference = 4.87157967212509e-1
n = 20 , left side = 8.09536658966781e-3
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.98995164947237e-1
n = 50 , left side = 3.49396872417307e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.05755525986819e-1
n = 100 , left side = 1.79071372384143e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.49397929427117e-1
n = 200 , left side = 9.06435452010634e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03122141884826e-1
n = 500 , left side = 3.65254093297129e-4
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54626644661537e-1

```

x

```

n = 10 , left side = 3.46541098802871e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.66533123746985e-1
n = 20 , left side = 1.73420259977516e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.89748505539153e-1
n = 50 , left side = 6.93681090501508e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.02312683805977e-1
n = 100 , left side = 3.46840545250782e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.47720237698450e-1
n = 200 , left side = 1.73420272625380e-3
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.02294374610583e-1
n = 500 , left side = 6.93681090501386e-4
        1/n^(3/10) = 1.54991898754834e-1

```


difference = 1.54298217664332e-1

$$x^2$$

n = 10 , left side = 4.73903023658121e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.53796931261460e-1
n = 20 , left side = 2.05308163937205e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.86559715143184e-1
n = 50 , left side = 7.44701748385268e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.01802477227139e-1
n = 100 , left side = 3.59595709721700e-3
1/n^(3/10) = 2.51188643150958e-1
difference = 2.47592686053741e-1
n = 200 , left side = 1.76609063743116e-3
1/n^(3/10) = 2.04028577336837e-1
difference = 2.02262486699406e-1
n = 500 , left side = 6.98783156289906e-4
1/n^(3/10) = 1.54991898754834e-1
difference = 1.54293115598544e-1

$$x^3$$

n = 10 , left side = 4.63238425467439e-2
1/n^(3/10) = 5.01187233627272e-1
difference = 4.54863391080528e-1
n = 20 , left side = 1.79448446203553e-2
1/n^(3/10) = 4.07090531536904e-1
difference = 3.89145686916549e-1
n = 50 , left side = 5.97784700290624e-3
1/n^(3/10) = 3.09249494710992e-1
difference = 3.03271647708085e-1
n = 100 , left side = 2.79387267593101e-3
1/n^(3/10) = 2.51188643150958e-1

```

        difference = 2.48394770475027e-1
n = 200 , left side = 1.34863905139215e-3
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.02679938285445e-1
n = 500 , left side = 5.27923845514616e-4
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54463974909319e-1

```

$$x^4$$

```

n = 10 , left side = 3.94349040023764e-2
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.61752329624896e-1
n = 20 , left side = 1.37990548305422e-2
        1/n^(3/10) = 4.07090531536904e-1
        difference = 3.93291476706362e-1
n = 50 , left side = 4.25445841424152e-3
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.04995036296750e-1
n = 100 , left side = 1.92806775614625e-3
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.49260575394812e-1
n = 200 , left side = 9.15246967505515e-4
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03113330369331e-1
n = 500 , left side = 3.54513590363287e-4
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54637385164470e-1

```

$$x^{10}$$

```

n = 10 , left side = 7.27638680160743e-3
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.93910846825665e-1
n = 20 , left side = 1.19062238971356e-3
        1/n^(3/10) = 4.07090531536904e-1

```

```

        difference = 4.05899909147191e-1
n = 50 , left side = 2.37641399649618e-4
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.09011853311342e-1
n = 100 , left side = 9.15159839724954e-5
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.51097127166986e-1
n = 200 , left side = 3.96334550487991e-5
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.03988943881788e-1
n = 500 , left side = 1.44549073568421e-5
        1/n^(3/10) = 1.54991898754834e-1
        difference = 1.54977443847477e-1

```

x0 = 1/2, Power = 3/10, lamda = 1, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 4.43886557706544e-3
        1/n^(3/10) = 5.01187233627272e-1
        difference = 4.96748368050207e-1
n = 20 , left side = 1.03630053646838e-3
        1/n^(3/10) = 4.07090531536904e-1
        difference = 4.06054231000436e-1
n = 50 , left side = 1.63624778015015e-4
        1/n^(3/10) = 3.09249494710992e-1
        difference = 3.09085869932977e-1
n = 100 , left side = 4.08320646757554e-5
        1/n^(3/10) = 2.51188643150958e-1
        difference = 2.51147811086282e-1
n = 200 , left side = 1.02034164284337e-5
        1/n^(3/10) = 2.04028577336837e-1
        difference = 2.04018373920409e-1

```

n = 500 , left side = 1.63234093675246e-6
 1/n^(3/10) = 1.54991898754834e-1
 difference = 1.54990266413897e-1

x

n = 10 , left side = 1.58663982219309e-5
 1/n^(3/10) = 5.01187233627272e-1
 difference = 5.01171367229050e-1
 n = 20 , left side = 6.52684128965575e-10
 1/n^(3/10) = 4.07090531536904e-1
 difference = 4.07090530884220e-1
 n = 50 , left side = 0.000000000000000e0
 1/n^(3/10) = 3.09249494710992e-1
 difference = 3.09249494710992e-1
 n = 100 , left side = 0.000000000000000e0
 1/n^(3/10) = 2.51188643150958e-1
 difference = 2.51188643150958e-1
 n = 200 , left side = 0.000000000000000e0
 1/n^(3/10) = 2.04028577336837e-1
 difference = 2.04028577336837e-1
 n = 500 , left side = 0.000000000000000e0
 1/n^(3/10) = 1.54991898754834e-1
 difference = 1.54991898754834e-1

x^2

n = 10 , left side = 1.15428308402608e-2
 1/n^(3/10) = 5.01187233627272e-1
 difference = 4.89644402787012e-1
 n = 20 , left side = 2.89205247195795e-3
 1/n^(3/10) = 4.07090531536904e-1
 difference = 4.04198479064946e-1
 n = 50 , left side = 4.62728557739434e-4
 1/n^(3/10) = 3.09249494710992e-1
 difference = 3.08786766153252e-1

$n = 100$, left side = 1.15682139434858e-4
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = 2.51072961011523e-1
 $n = 200$, left side = 2.89205348588673e-5
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = 2.03999656801978e-1
 $n = 500$, left side = 4.62728557748537e-6
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = 1.54987271469256e-1

$$x^3$$

$n = 10$, left side = 1.73200399632407e-2
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = 4.83867193664032e-1
 $n = 20$, left side = 4.33807899343377e-3
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = 4.02752452543471e-1
 $n = 50$, left side = 6.94092836609345e-4
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = 3.08555401874382e-1
 $n = 100$, left side = 1.73523209152371e-4
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = 2.51015119941806e-1
 $n = 200$, left side = 4.33808022881066e-5
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = 2.03985196534549e-1
 $n = 500$, left side = 6.94092836614479e-6
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = 1.54984957826468e-1

$$x^4$$

$n = 10$, left side = 1.77823913197880e-2
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = 4.83404842307484e-1

$n = 20$, left side = 4.36734781351719e-3
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = 4.02723183723387e-1
 $n = 50$, left side = 6.94842122375819e-4
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = 3.08554652588616e-1
 $n = 100$, left side = 1.73570039512738e-4
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = 2.51015073111445e-1
 $n = 200$, left side = 4.33837291856087e-5
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = 2.03985193607651e-1
 $n = 500$, left side = 6.94100329461100e-6
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = 1.54984957751539e-1

$$x^{10}$$

$n = 10$, left side = 4.06239089134456e-3
 $1/n^{(3/10)} = 5.01187233627272e-1$
difference = 4.97124842735928e-1
 $n = 20$, left side = 6.12650024829540e-4
 $1/n^{(3/10)} = 4.07090531536904e-1$
difference = 4.06477881512075e-1
 $n = 50$, left side = 8.38305099970637e-5
 $1/n^{(3/10)} = 3.09249494710992e-1$
difference = 3.09165664200995e-1
 $n = 100$, left side = 2.04889255338266e-5
 $1/n^{(3/10)} = 2.51188643150958e-1$
difference = 2.51168154225424e-1
 $n = 200$, left side = 5.09329964822080e-6
 $1/n^{(3/10)} = 2.04028577336837e-1$
difference = 2.04023484037189e-1
 $n = 500$, left side = 8.13635935028711e-7
 $1/n^{(3/10)} = 1.54991898754834e-1$
difference = 1.54991085118899e-1

x0 = 1/2, Power = 1/2, lamda = 1/4, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 4.65939350634126e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.69633830953425e-1
n = 20 , left side = 5.39175273339064e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.69689270416073e-1
n = 50 , left side = 2.66706958509516e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.14750660386358e-1
n = 100 , left side = 1.39692422376505e-2
          1/n^(1/2) = 1.00000000000000e-1
          difference = 8.60307577623495e-2
n = 200 , left side = 7.15466879542836e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.35560093232264e-2
n = 500 , left side = 2.90466167844949e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.18166978715463e-2

```

$$x$$

```

n = 10 , left side = 1.46960541508878e-1
          1/n^(1/2) = 3.16227766016838e-1
          difference = 1.69267224507960e-1
n = 20 , left side = 1.28044818944893e-1
          1/n^(1/2) = 2.23606797749979e-1
          difference = 9.55619788050856e-2
n = 50 , left side = 5.54458428059442e-2

```

$1/n^{(1/2)} = 1.41421356237310e-1$
 difference = 8.59755134313653e-2
 n = 100 , left side = 2.77258872001896e-2
 $1/n^{(1/2)} = 1.00000000000000e-1$
 difference = 7.22741127998104e-2
 n = 200 , left side = 1.38629436111991e-2
 $1/n^{(1/2)} = 7.07106781186548e-2$
 difference = 5.68477345074556e-2
 n = 500 , left side = 5.54517744447935e-3
 $1/n^{(1/2)} = 4.47213595499958e-2$
 difference = 3.91761821055164e-2

$$x^2$$

n = 10 , left side = 2.44068957215094e-1
 $1/n^{(1/2)} = 3.16227766016838e-1$
 difference = 7.21588088017442e-2
 n = 20 , left side = 1.73393496362237e-1
 $1/n^{(1/2)} = 2.23606797749979e-1$
 difference = 5.02133013877419e-2
 n = 50 , left side = 6.39143140576404e-2
 $1/n^{(1/2)} = 1.41421356237310e-1$
 difference = 7.75070421796691e-2
 n = 100 , left side = 2.98438925970291e-2
 $1/n^{(1/2)} = 1.00000000000000e-1$
 difference = 7.01561074029709e-2
 n = 200 , left side = 1.43924449634693e-2
 $1/n^{(1/2)} = 7.07106781186548e-2$
 difference = 5.63182331551854e-2
 n = 500 , left side = 5.62989766084276e-3
 $1/n^{(1/2)} = 4.47213595499958e-2$
 difference = 3.90914618891530e-2

$$x^3$$

n = 10 , left side = 2.70532081486956e-1


```

1/n^(1/2) = 3.16227766016838e-1
difference = 4.56956845298823e-2
n = 20 , left side = 1.75303858260733e-1
1/n^(1/2) = 2.23606797749979e-1
difference = 4.83029394892459e-2
n = 50 , left side = 5.53534166881951e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 8.60679395491144e-2
n = 100 , left side = 2.41049670707777e-2
1/n^(1/2) = 1.00000000000000e-1
difference = 7.58950329292223e-2
n = 200 , left side = 1.12081526845386e-2
1/n^(1/2) = 7.07106781186548e-2
difference = 5.95025254341161e-2
n = 500 , left side = 4.28703175655951e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.04343277934363e-2

```

$$x^4$$

```

n = 10 , left side = 2.71811655321145e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 4.44161106956934e-2
n = 20 , left side = 1.59700780897926e-1
1/n^(1/2) = 2.23606797749979e-1
difference = 6.39060168520534e-2
n = 50 , left side = 4.27867893894554e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 9.86345668478541e-2
n = 100 , left side = 1.73213915349858e-2
1/n^(1/2) = 1.00000000000000e-1
difference = 8.26786084650142e-2
n = 200 , left side = 7.76000677274204e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.29506713459127e-2
n = 500 , left side = 2.90182870840223e-3

```

$1/n^{(1/2)} = 4.47213595499958e-2$
 $\text{difference} = 4.18195308415936e-2$

$$x^{10}$$

$n = 10$, left side = $1.86850823331552e-1$
 $1/n^{(1/2)} = 3.16227766016838e-1$
 $\text{difference} = 1.29376942685286e-1$
 $n = 20$, left side = $6.58347789205075e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$
 $\text{difference} = 1.57772018829471e-1$
 $n = 50$, left side = $4.87085072187464e-3$
 $1/n^{(1/2)} = 1.41421356237310e-1$
 $\text{difference} = 1.36550505515435e-1$
 $n = 100$, left side = $1.09970741430996e-3$
 $1/n^{(1/2)} = 1.00000000000000e-1$
 $\text{difference} = 9.89002925856901e-2$
 $n = 200$, left side = $3.82804893814586e-4$
 $1/n^{(1/2)} = 7.07106781186548e-2$
 $\text{difference} = 7.03278732248402e-2$
 $n = 500$, left side = $1.24276968148264e-4$
 $1/n^{(1/2)} = 4.47213595499958e-2$
 $\text{difference} = 4.45970825818475e-2$

 $x_0 = 1/2$, Power = $1/2$, lamda = $1/4$, $q = 1/2$

$$x^{\frac{1}{3}}$$

$n = 10$, left side = $1.96080052578450e-2$
 $1/n^{(1/2)} = 3.16227766016838e-1$
 $\text{difference} = 2.96619760758993e-1$
 $n = 20$, left side = $2.08299179386359e-2$
 $1/n^{(1/2)} = 2.23606797749979e-1$

```

        difference = 2.02776879811343e-1
n = 50 , left side = 1.26071386109592e-2
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.28814217626350e-1
n = 100 , left side = 6.81015763376169e-3
        1/n^(1/2) = 1.00000000000000e-1
        difference = 9.31898423662383e-2
n = 200 , left side = 3.53432821873112e-3
        1/n^(1/2) = 7.07106781186548e-2
        difference = 6.71763498999236e-2
n = 500 , left side = 1.44549650366488e-3
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.32758630463309e-2

```

x

```

n = 10 , left side = 5.68097172543791e-2
        1/n^(1/2) = 3.16227766016838e-1
        difference = 2.59418048762459e-1
n = 20 , left side = 6.32752819000137e-2
        1/n^(1/2) = 2.23606797749979e-1
        difference = 1.60331515849965e-1
n = 50 , left side = 2.77227554082915e-2
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.13698600829018e-1
n = 100 , left side = 1.38629435997853e-2
        1/n^(1/2) = 1.00000000000000e-1
        difference = 8.61370564002147e-2
n = 200 , left side = 6.93147180559972e-3
        1/n^(1/2) = 7.07106781186548e-2
        difference = 6.37792063130550e-2
n = 500 , left side = 2.77258872223995e-3
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.19487708277558e-2

```

x^2

$n = 10$, left side = 1.47872752410878e-1
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 1.68355013605960e-1
 $n = 20$, left side = 9.78605569036922e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 1.25746240846287e-1
 $n = 50$, left side = 3.38868135786705e-2
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.07534542658639e-1
 $n = 100$, left side = 1.54044053860309e-2
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 8.45955946139691e-2
 $n = 200$, left side = 7.31683725369431e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.33938408649604e-2
 $n = 500$, left side = 2.83424719393494e-3
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.18871123560609e-2

$$x^3$$

$n = 10$, left side = 1.73192000431426e-1
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 1.43035765585412e-1
 $n = 20$, left side = 1.03864925521532e-1
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 1.19741872228447e-1
 $n = 50$, left side = 3.05073733511566e-2
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.10913982886153e-1
 $n = 100$, left side = 1.27681795752464e-2
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 8.72318204247536e-2
 $n = 200$, left side = 5.78399942625288e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.49266786924019e-2

n = 500 , left side = 2.17239948281706e-3
 1/n^(1/2) = 4.47213595499958e-2
 difference = 4.25489600671787e-2

$$x^4$$

n = 10 , left side = 1.80257354696213e-1
 1/n^(1/2) = 3.16227766016838e-1
 difference = 1.35970411320625e-1
 n = 20 , left side = 9.63498090064417e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 1.27256988743537e-1
 n = 50 , left side = 2.41914336230855e-2
 1/n^(1/2) = 1.41421356237310e-1
 difference = 1.17229922614224e-1
 n = 100 , left side = 9.37035404303910e-3
 1/n^(1/2) = 1.00000000000000e-1
 difference = 9.06296459569609e-2
 n = 200 , left side = 4.05904957310241e-3
 1/n^(1/2) = 7.07106781186548e-2
 difference = 6.66516285455523e-2
 n = 500 , left side = 1.47973714572865e-3
 1/n^(1/2) = 4.47213595499958e-2
 difference = 4.32416224042671e-2

$$x^{10}$$

n = 10 , left side = 1.19260700953333e-1
 1/n^(1/2) = 3.16227766016838e-1
 difference = 1.96967065063505e-1
 n = 20 , left side = 3.73465208237525e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 1.86260276926226e-1
 n = 50 , left side = 2.80162956157206e-3
 1/n^(1/2) = 1.41421356237310e-1
 difference = 1.38619726675737e-1

```

n = 100 , left side = 6.32905221145845e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.93670947788542e-2
n = 200 , left side = 2.12043953336508e-4
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.04986341653182e-2
n = 500 , left side = 6.54808182457762e-5
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.46558787317500e-2

```

x0 = 1/2, Power = 1/2, lamda = 1/4, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 8.10134985303516e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.35214267486486e-1
n = 20 , left side = 1.68560763315217e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.06750721418457e-1
n = 50 , left side = 1.97802106438832e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.39443335172921e-1
n = 100 , left side = 4.80026592954430e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.95199734070456e-2
n = 200 , left side = 1.19239646471425e-4
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.05914384721833e-2
n = 500 , left side = 1.90449720897101e-5
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47023145779061e-2

```

x

```
n = 10 , left side = 4.91637605442458e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.67064005472592e-1
n = 20 , left side = 3.39394250680042e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.20212855243179e-1
n = 50 , left side = 1.64890433890230e-6
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.41419707332971e-1
n = 100 , left side = 5.86114490275236e-12
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.99999999941389e-2
n = 200 , left side = 0.00000000000000e0
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.07106781186548e-2
n = 500 , left side = 0.00000000000000e0
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47213595499958e-2
```

x^2

```
n = 10 , left side = 5.25230782287648e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.63704687788073e-1
n = 20 , left side = 2.83373415030339e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.95269456246945e-1
n = 50 , left side = 5.39457664221732e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.36026779595092e-1
n = 100 , left side = 1.34928057788158e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.86507194221184e-2
n = 200 , left side = 3.37320146703013e-4
```

```

1/n^(1/2) = 7.07106781186548e-2
difference = 7.03733579719517e-2
n = 500 , left side = 5.39712234724488e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46673883265233e-2

```

$$x^3$$

```

n = 10 , left side = 8.22234506283697e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.34004315388468e-1
n = 20 , left side = 4.36057626242193e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.80001035125760e-1
n = 50 , left side = 8.09259320656272e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.33328763030747e-1
n = 100 , left side = 2.02392086959333e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.79760791304067e-2
n = 200 , left side = 5.05980220054492e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.02046978986003e-2
n = 500 , left side = 8.09568352087009e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46404027147871e-2

```

$$x^4$$

```

n = 10 , left side = 1.01052973293562e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 2.15174792723276e-1
n = 20 , left side = 4.67481413489399e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.76858656401039e-1
n = 50 , left side = 8.21282675171919e-3

```



```

1/n^(1/2) = 1.41421356237310e-1
difference = 1.33208529485590e-1
n = 100 , left side = 2.03145927015694e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.79685407298431e-2
n = 200 , left side = 5.06451370153385e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.02042267485014e-2
n = 500 , left side = 8.09688966512678e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46403906533445e-2

```

$$x^{10}$$

```

n = 10 , left side = 7.01207124827065e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.46107053534131e-1
n = 20 , left side = 2.01574704246336e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.03449327325345e-1
n = 50 , left side = 1.42480124701184e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.39996554990298e-1
n = 100 , left side = 2.63178146244861e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.97368218537551e-2
n = 200 , left side = 6.08599933529959e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06498181253018e-2
n = 500 , left side = 9.52678518000342e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47118327648158e-2

```

$x_0 = 1/2$, Power = $1/2$, lamda = $1/2$, $q = 1/4$

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 5.33369660315088e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.62890799985329e-1
n = 20 , left side = 3.23728371506258e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.91233960599353e-1
n = 50 , left side = 1.39366391997124e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.27484717037597e-1
n = 100 , left side = 7.14621856758857e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.28537814324114e-2
n = 200 , left side = 3.61961230894126e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.70910658097135e-2
n = 500 , left side = 1.45930076253686e-3
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.32620587874589e-2

```

$$x$$

```

n = 10 , left side = 1.29071817455777e-1
          1/n^(1/2) = 3.16227766016838e-1
          difference = 1.87155948561061e-1
n = 20 , left side = 6.92514035886481e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.54355394161331e-1
n = 50 , left side = 2.77258876437841e-2
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.13695468593525e-1
n = 100 , left side = 1.38629438314646e-2
          1/n^(1/2) = 1.00000000000000e-1

```

```

        difference = 8.61370561685354e-2
n = 200 , left side = 6.93147191573251e-3
        1/n^(1/2) = 7.07106781186548e-2
        difference = 6.37792062029222e-2
n = 500 , left side = 2.77258876629327e-3
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.19487707837025e-2

```

$$x^2$$

```

n = 10 , left side = 1.77390913518807e-1
        1/n^(1/2) = 3.16227766016838e-1
        difference = 1.38836852498031e-1
n = 20 , left side = 8.30737213816240e-2
        1/n^(1/2) = 2.23606797749979e-1
        difference = 1.40533076368355e-1
n = 50 , left side = 2.99438930027937e-2
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.11477463234516e-1
n = 100 , left side = 1.44174451738843e-2
        1/n^(1/2) = 1.00000000000000e-1
        difference = 8.55825548261157e-2
n = 200 , left side = 7.07009725133750e-3
        1/n^(1/2) = 7.07106781186548e-2
        difference = 6.36405808673172e-2
n = 500 , left side = 2.79476881998975e-3
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.19265907300060e-2

```

$$x^3$$

```

n = 10 , left side = 1.81706840017598e-1
        1/n^(1/2) = 3.16227766016838e-1
        difference = 1.34520925999240e-1
n = 20 , left side = 7.48642399521749e-2
        1/n^(1/2) = 2.23606797749979e-1

```

```

        difference = 1.48742557797804e-1
n = 50 , left side = 2.42632851043290e-2
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.17158071132980e-1
n = 100 , left side = 1.12466925545658e-2
        1/n^(1/2) = 1.00000000000000e-1
        difference = 8.87533074454342e-2
n = 200 , left side = 5.40875852362388e-3
        1/n^(1/2) = 7.07106781186548e-2
        difference = 6.53019195950309e-2
n = 500 , left side = 2.11285351660367e-3
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.26085060333921e-2

```

$$x^4$$

```

n = 10 , left side = 1.67787178069694e-1
        1/n^(1/2) = 3.16227766016838e-1
        difference = 1.48440587947144e-1
n = 20 , left side = 6.03397370785075e-2
        1/n^(1/2) = 2.23606797749979e-1
        difference = 1.63267060671471e-1
n = 50 , left side = 1.74893080184918e-2
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.23932048218818e-1
n = 100 , left side = 7.79966635748697e-3
        1/n^(1/2) = 1.00000000000000e-1
        difference = 9.22003336425130e-2
n = 200 , left side = 3.67816819644864e-3
        1/n^(1/2) = 7.07106781186548e-2
        difference = 6.70325099222061e-2
n = 500 , left side = 1.41984974971847e-3
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.33015098002773e-2

```

$$x^{10}$$

```

n = 10 , left side = 7.68027797084298e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.39424986308408e-1
n = 20 , left side = 9.76146773173466e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.13845330018244e-1
n = 50 , left side = 1.13068528957820e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.40290670947731e-1
n = 100 , left side = 3.88474877995726e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.96115251220043e-2
n = 200 , left side = 1.62038602246220e-4
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.05486395164085e-2
n = 500 , left side = 5.81892063837631e-5
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.46631703436120e-2

```

x0 = 1/2, Power = 1/2, lamda = 1/2, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.96445247579275e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.96583241258910e-1
n = 20 , left side = 1.48888789279412e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.08717918822038e-1
n = 50 , left side = 6.77605354912070e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.34645302688189e-1
n = 100 , left side = 3.52568356510685e-3

```

```

1/n^(1/2) = 1.000000000000000e-1
difference = 9.64743164348932e-2
n = 200 , left side = 1.79801315131500e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.89126649673397e-2
n = 500 , left side = 7.27768804897111e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.39935907450987e-2

```

x

```

n = 10 , left side = 6.38935662496244e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.52334199767214e-1
n = 20 , left side = 3.46235470985199e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.88983250651459e-1
n = 50 , left side = 1.38629429714340e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.27558413265876e-1
n = 100 , left side = 6.93147149063511e-3
1/n^(1/2) = 1.000000000000000e-1
difference = 9.30685285093649e-2
n = 200 , left side = 3.46573574531772e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.72449423733370e-2
n = 500 , left side = 1.38629429812720e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.33350652518686e-2

```

x^2

```

n = 10 , left side = 1.01231195168683e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 2.14996570848155e-1
n = 20 , left side = 4.48623512045705e-2

```

```

1/n^(1/2) = 2.23606797749979e-1
difference = 1.78744446545408e-1
n = 50 , left side = 1.55044047114581e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.25916951525851e-1
n = 100 , left side = 7.34183692697737e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.26581630730226e-2
n = 200 , left side = 3.56832710440319e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.71423510142516e-2
n = 500 , left side = 1.40270891558075e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.33186506344150e-2

```

$$x^3$$

```

n = 10 , left side = 1.09051180141690e-1
1/n^(1/2) = 3.16227766016838e-1
difference = 2.07176585875148e-1
n = 20 , left side = 4.22970110547811e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 1.81309786695198e-1
n = 50 , left side = 1.29223379237350e-2
1/n^(1/2) = 1.41421356237310e-1
difference = 1.28499018313574e-1
n = 100 , left side = 5.82201903348833e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.41779809665117e-2
n = 200 , left side = 2.75417225524147e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.79565058634133e-2
n = 500 , left side = 1.06440558786389e-3
1/n^(1/2) = 4.47213595499958e-2
difference = 4.36569539621319e-2

```

$$x^4$$

```

n = 10 , left side = 1.02540884681400e-1
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.13686881335438e-1
n = 20 , left side = 3.49981874860216e-2
          1/n^(1/2) = 2.23606797749979e-1
          difference = 1.88608610263957e-1
n = 50 , left side = 9.52960631552958e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.31891749921780e-1
n = 100 , left side = 4.09764755001929e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.59023524499807e-2
n = 200 , left side = 1.88876104704880e-3
          1/n^(1/2) = 7.07106781186548e-2
          difference = 6.88219170716060e-2
n = 500 , left side = 7.17895958025183e-4
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.40034635919706e-2

```

$$x^{10}$$

```

n = 10 , left side = 4.39432749437958e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 2.72284491073042e-1
n = 20 , left side = 5.59881198565948e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.18007985764319e-1
n = 50 , left side = 6.58122885628435e-4
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.40763233351681e-1
n = 100 , left side = 2.17135216419696e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.97828647835803e-2
n = 200 , left side = 8.67799400496487e-5

```



```

1/n^(1/2) = 7.07106781186548e-2
difference = 7.06238981786051e-2
n = 500 , left side = 3.00238003038830e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46913357496919e-2

```

x0 = 1/2, Power = 1/2, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.88583358245119e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.97369430192326e-1
n = 20 , left side = 3.43036193768762e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.20176435812291e-1
n = 50 , left side = 5.15745217625541e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.40905611019684e-1
n = 100 , left side = 1.28085535782807e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.98719144642172e-2
n = 200 , left side = 3.19697272262998e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06787083914285e-2
n = 500 , left side = 5.11285838666886e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47162466916091e-2

```

$$x$$

```

n = 10 , left side = 3.03097878388953e-3
1/n^(1/2) = 3.16227766016838e-1

```

```

        difference = 3.13196787232948e-1
n = 20 , left side = 1.79813965206499e-5
        1/n^(1/2) = 2.23606797749979e-1
        difference = 2.23588816353458e-1
n = 50 , left side = 5.04973840520506e-12
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.41421356232260e-1
n = 100 , left side = 1.11022302462516e-16
        1/n^(1/2) = 1.00000000000000e-1
        difference = 9.99999999999999e-2
n = 200 , left side = 0.00000000000000e0
        1/n^(1/2) = 7.07106781186548e-2
        difference = 7.07106781186548e-2
n = 500 , left side = 5.55111512312578e-17
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.47213595499957e-2

```

$$x^2$$

```

n = 10 , left side = 3.13288100022530e-2
        1/n^(1/2) = 3.16227766016838e-1
        difference = 2.84898956014585e-1
n = 20 , left side = 9.02981956494553e-3
        1/n^(1/2) = 2.23606797749979e-1
        difference = 2.14576978185033e-1
n = 50 , left side = 1.44928066358080e-3
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.39972075573729e-1
n = 100 , left side = 3.62320167826280e-4
        1/n^(1/2) = 1.00000000000000e-1
        difference = 9.96376798321737e-2
n = 200 , left side = 9.05800419565561e-5
        1/n^(1/2) = 7.07106781186548e-2
        difference = 7.06200980766982e-2
n = 500 , left side = 1.44928067130623e-5
        1/n^(1/2) = 4.47213595499958e-2

```

difference = 4.47068667432827e-2

$$x^3$$

n = 10 , left side = 4.78600213916086e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.68367744625229e-1
n = 20 , left side = 1.35520533206201e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.10054744429359e-1
n = 50 , left side = 2.17392099772531e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.39247435239584e-1
n = 100 , left side = 5.43480251739309e-4
1/n^(1/2) = 1.00000000000000e-1
difference = 9.94565197482607e-2
n = 200 , left side = 1.35870062934806e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.05748080557199e-2
n = 500 , left side = 2.17392100695102e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46996203399263e-2

$$x^4$$

n = 10 , left side = 5.14533906505907e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.64774375366247e-1
n = 20 , left side = 1.38732244570932e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.09733573292886e-1
n = 50 , left side = 2.18227896612364e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.39239077271186e-1
n = 100 , left side = 5.44002624822695e-4
1/n^(1/2) = 1.00000000000000e-1

```

difference = 9.94559973751773e-2
n = 200 , left side = 1.35902711252578e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 7.05747754074022e-2
n = 500 , left side = 2.17400458664835e-5
1/n^(1/2) = 4.47213595499958e-2
difference = 4.46996195041293e-2

```

$$x^{10}$$

```

n = 10 , left side = 2.39574413183785e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.92270324698459e-1
n = 20 , left side = 2.97086004062642e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.20635937709353e-1
n = 50 , left side = 2.83599622557975e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41137756614752e-1
n = 100 , left side = 6.54249762594573e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99345750237405e-2
n = 200 , left side = 1.60297404174447e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06946483782373e-2
n = 500 , left side = 2.55030753047421e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47188092424653e-2

```

x0 = 1/2, Power = 1/2, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

$n = 10$, left side = 3.15884348850566e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.84639331131781e-1
 $n = 20$, left side = 1.69988381319672e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 2.06607959618012e-1
 $n = 50$, left side = 7.10929624048973e-3
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.34312059996820e-1
 $n = 100$, left side = 3.60940309324842e-3
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.63905969067516e-2
 $n = 200$, left side = 1.81874048780928e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.88919376308455e-2
 $n = 500$, left side = 7.30913664388266e-4
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.39904458856075e-2

x

$n = 10$, left side = 6.92279510900182e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.46999814926820e-1
 $n = 20$, left side = 3.46421335720477e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 1.88964664177931e-1
 $n = 50$, left side = 1.38568544082612e-2
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.27564501829048e-1
 $n = 100$, left side = 6.92842720413067e-3
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.30715727958693e-2
 $n = 200$, left side = 3.46421360206539e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.72464645165894e-2

n = 500 , left side = 1.38568544082618e-3
 1/n^(1/2) = 4.47213595499958e-2
 difference = 4.33356741091696e-2

$$x^2$$

n = 10 , left side = 8.55451218222509e-2
 1/n^(1/2) = 3.16227766016838e-1
 difference = 2.30682644194587e-1
 n = 20 , left side = 3.87308178004870e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 1.84875979949492e-1
 n = 50 , left side = 1.45110441151024e-2
 1/n^(1/2) = 1.41421356237310e-1
 difference = 1.26910312122207e-1
 n = 100 , left side = 7.09197463084094e-3
 1/n^(1/2) = 1.00000000000000e-1
 difference = 9.29080253691591e-2
 n = 200 , left side = 3.50510045874292e-3
 1/n^(1/2) = 7.07106781186548e-2
 difference = 6.72055776599118e-2
 n = 500 , left side = 1.39222733789462e-3
 1/n^(1/2) = 4.47213595499958e-2
 difference = 4.33291322121012e-2

$$x^3$$

n = 10 , left side = 7.91097899306038e-2
 1/n^(1/2) = 3.16227766016838e-1
 difference = 2.37117976086234e-1
 n = 20 , left side = 3.24567771921774e-2
 1/n^(1/2) = 2.23606797749979e-1
 difference = 1.91150020557802e-1
 n = 50 , left side = 1.13958230614945e-2
 1/n^(1/2) = 1.41421356237310e-1
 difference = 1.30025533175815e-1

$n = 100$, left side = 5.44437875504283e-3
 $1/n^{(1/2)} = 1.000000000000000e-1$
difference = 9.45556212449572e-2
 $n = 200$, left side = 2.65983263805031e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.80508454806044e-2
 $n = 500$, left side = 1.04909882391729e-3
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.36722607260785e-2

$$x^4$$

$n = 10$, left side = 6.53260321168637e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.50901733899974e-1
 $n = 20$, left side = 2.41899798353885e-2
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 1.99416817914590e-1
 $n = 50$, left side = 7.95482775153313e-3
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.33466528485776e-1
 $n = 100$, left side = 3.71509170320668e-3
 $1/n^{(1/2)} = 1.000000000000000e-1$
difference = 9.62849082967933e-2
 $n = 200$, left side = 1.79412654760111e-3
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 6.89165515710536e-2
 $n = 500$, left side = 7.02699493465395e-4
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.40186600565304e-2

$$x^{10}$$

$n = 10$, left side = 1.23005431860469e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 3.03927222830791e-1

```

n = 20 , left side = 1.95443229260425e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.21652365457375e-1
n = 50 , left side = 4.11078488591595e-4
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.41010277748718e-1
n = 100 , left side = 1.66923378278077e-4
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.98330766217219e-2
n = 200 , left side = 7.51857729315232e-5
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.06354923457232e-2
n = 500 , left side = 2.82350791906388e-5
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.46931244708052e-2

```

x0 = 1/2, Power = 1/2, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.40292664147630e-2
          1/n^(1/2) = 3.16227766016838e-1
          difference = 3.02198499602075e-1
n = 20 , left side = 8.09536658966781e-3
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.15511431160311e-1
n = 50 , left side = 3.49396872417307e-3
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.37927387513136e-1
n = 100 , left side = 1.79071372384143e-3
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.82092862761586e-2
n = 200 , left side = 9.06435452010634e-4

```



```

1/n^(1/2) = 7.07106781186548e-2
difference = 6.98042426666441e-2
n = 500 , left side = 3.65254093297129e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.43561054566987e-2

```

x

```

n = 10 , left side = 3.46541098802871e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.81573656136551e-1
n = 20 , left side = 1.73420259977516e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.06264771752227e-1
n = 50 , left side = 6.93681090501508e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.34484545332294e-1
n = 100 , left side = 3.46840545250782e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.65315945474922e-2
n = 200 , left side = 1.73420272625380e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.89764753924010e-2
n = 500 , left side = 6.93681090501386e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.40276784594944e-2

```

x^2

```

n = 10 , left side = 4.73903023658121e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.68837463651026e-1
n = 20 , left side = 2.05308163937205e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.03075981356259e-1
n = 50 , left side = 7.44701748385268e-3

```

```

1/n^(1/2) = 1.41421356237310e-1
difference = 1.33974338753457e-1
n = 100 , left side = 3.59595709721700e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.64040429027830e-2
n = 200 , left side = 1.76609063743116e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.89445874812236e-2
n = 500 , left side = 6.98783156289906e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.40225763937059e-2

```

$$x^3$$

```

n = 10 , left side = 4.63238425467439e-2
1/n^(1/2) = 3.16227766016838e-1
difference = 2.69903923470094e-1
n = 20 , left side = 1.79448446203553e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.05661953129624e-1
n = 50 , left side = 5.97784700290624e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.35443509234403e-1
n = 100 , left side = 2.79387267593101e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.72061273240690e-2
n = 200 , left side = 1.34863905139215e-3
1/n^(1/2) = 7.07106781186548e-2
difference = 6.93620390672626e-2
n = 500 , left side = 5.27923845514616e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.41934357044812e-2

```

$$x^4$$

```

n = 10 , left side = 3.94349040023764e-2

```

```

1/n^(1/2) = 3.16227766016838e-1
difference = 2.76792862014462e-1
n = 20 , left side = 1.37990548305422e-2
1/n^(1/2) = 2.23606797749979e-1
difference = 2.09807742919437e-1
n = 50 , left side = 4.25445841424152e-3
1/n^(1/2) = 1.41421356237310e-1
difference = 1.37166897823068e-1
n = 100 , left side = 1.92806775614625e-3
1/n^(1/2) = 1.00000000000000e-1
difference = 9.80719322438538e-2
n = 200 , left side = 9.15246967505515e-4
1/n^(1/2) = 7.07106781186548e-2
difference = 6.97954311511492e-2
n = 500 , left side = 3.54513590363287e-4
1/n^(1/2) = 4.47213595499958e-2
difference = 4.43668459596325e-2

```

$$x^{10}$$

```

n = 10 , left side = 7.27638680160743e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.08951379215231e-1
n = 20 , left side = 1.19062238971356e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.22416175360265e-1
n = 50 , left side = 2.37641399649618e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41183714837660e-1
n = 100 , left side = 9.15159839724954e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99084840160275e-2
n = 200 , left side = 3.96334550487991e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.06710446636060e-2
n = 500 , left side = 1.44549073568421e-5

```

1/n^(1/2) = 4.47213595499958e-2
difference = 4.47069046426390e-2

x0 = 1/2, Power = 1/2, lamda = 1, q = 1

$$x^{\frac{1}{3}}$$

n = 10 , left side = 4.43886557706544e-3
1/n^(1/2) = 3.16227766016838e-1
difference = 3.11788900439773e-1
n = 20 , left side = 1.03630053646838e-3
1/n^(1/2) = 2.23606797749979e-1
difference = 2.22570497213511e-1
n = 50 , left side = 1.63624778015015e-4
1/n^(1/2) = 1.41421356237310e-1
difference = 1.41257731459294e-1
n = 100 , left side = 4.08320646757554e-5
1/n^(1/2) = 1.00000000000000e-1
difference = 9.99591679353243e-2
n = 200 , left side = 1.02034164284337e-5
1/n^(1/2) = 7.07106781186548e-2
difference = 7.07004747022263e-2
n = 500 , left side = 1.63234093675246e-6
1/n^(1/2) = 4.47213595499958e-2
difference = 4.47197272090590e-2

$$x$$

n = 10 , left side = 1.58663982219309e-5
1/n^(1/2) = 3.16227766016838e-1
difference = 3.16211899618616e-1
n = 20 , left side = 6.52684128965575e-10
1/n^(1/2) = 2.23606797749979e-1

```

        difference = 2.23606797097295e-1
n = 50 , left side = 0.000000000000000e0
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.41421356237310e-1
n = 100 , left side = 0.000000000000000e0
        1/n^(1/2) = 1.000000000000000e-1
        difference = 1.000000000000000e-1
n = 200 , left side = 0.000000000000000e0
        1/n^(1/2) = 7.07106781186548e-2
        difference = 7.07106781186548e-2
n = 500 , left side = 0.000000000000000e0
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.47213595499958e-2

```

$$x^2$$

```

n = 10 , left side = 1.15428308402608e-2
        1/n^(1/2) = 3.16227766016838e-1
        difference = 3.04684935176577e-1
n = 20 , left side = 2.89205247195795e-3
        1/n^(1/2) = 2.23606797749979e-1
        difference = 2.20714745278021e-1
n = 50 , left side = 4.62728557739434e-4
        1/n^(1/2) = 1.41421356237310e-1
        difference = 1.40958627679570e-1
n = 100 , left side = 1.15682139434858e-4
        1/n^(1/2) = 1.000000000000000e-1
        difference = 9.98843178605651e-2
n = 200 , left side = 2.89205348588673e-5
        1/n^(1/2) = 7.07106781186548e-2
        difference = 7.06817575837959e-2
n = 500 , left side = 4.62728557748537e-6
        1/n^(1/2) = 4.47213595499958e-2
        difference = 4.47167322644183e-2

```

$$x^3$$

$n = 10$, left side = 1.73200399632407e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.98907726053597e-1
 $n = 20$, left side = 4.33807899343377e-3
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 2.19268718756545e-1
 $n = 50$, left side = 6.94092836609345e-4
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.40727263400700e-1
 $n = 100$, left side = 1.73523209152371e-4
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.98264767908476e-2
 $n = 200$, left side = 4.33808022881066e-5
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 7.06672973163666e-2
 $n = 500$, left side = 6.94092836614479e-6
 $1/n^{(1/2)} = 4.47213595499958e-2$
difference = 4.47144186216296e-2

$$x^4$$

$n = 10$, left side = 1.77823913197880e-2
 $1/n^{(1/2)} = 3.16227766016838e-1$
difference = 2.98445374697050e-1
 $n = 20$, left side = 4.36734781351719e-3
 $1/n^{(1/2)} = 2.23606797749979e-1$
difference = 2.19239449936462e-1
 $n = 50$, left side = 6.94842122375819e-4
 $1/n^{(1/2)} = 1.41421356237310e-1$
difference = 1.40726514114934e-1
 $n = 100$, left side = 1.73570039512738e-4
 $1/n^{(1/2)} = 1.00000000000000e-1$
difference = 9.98264299604873e-2
 $n = 200$, left side = 4.33837291856087e-5
 $1/n^{(1/2)} = 7.07106781186548e-2$
difference = 7.06672943894691e-2

```

n = 500 , left side = 6.94100329461100e-6
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47144185467012e-2

```

$$x^{10}$$

```

n = 10 , left side = 4.06239089134456e-3
          1/n^(1/2) = 3.16227766016838e-1
          difference = 3.12165375125493e-1
n = 20 , left side = 6.12650024829540e-4
          1/n^(1/2) = 2.23606797749979e-1
          difference = 2.22994147725149e-1
n = 50 , left side = 8.38305099970637e-5
          1/n^(1/2) = 1.41421356237310e-1
          difference = 1.41337525727312e-1
n = 100 , left side = 2.04889255338266e-5
          1/n^(1/2) = 1.00000000000000e-1
          difference = 9.99795110744662e-2
n = 200 , left side = 5.09329964822080e-6
          1/n^(1/2) = 7.07106781186548e-2
          difference = 7.07055848190065e-2
n = 500 , left side = 8.13635935028711e-7
          1/n^(1/2) = 4.47213595499958e-2
          difference = 4.47205459140608e-2

```

x0 = 1/2, Power = 7/10, lamda = 1/4, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 4.65939350634126e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.52932296433475e-1
n = 20 , left side = 5.39175273339064e-2

```

```

1/n^(7/10) = 1.22822802611579e-1
difference = 6.89052752776726e-2
n = 50 , left side = 2.66706958509516e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 3.80020048067841e-2
n = 100 , left side = 1.39692422376505e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 2.58414748176992e-2
n = 200 , left side = 7.15466879542836e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.73517021515461e-2
n = 500 , left side = 2.90466167844949e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 9.99923856451483e-3

```

x

```

n = 10 , left side = 1.46960541508878e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 5.25656899880098e-2
n = 20 , left side = 1.28044818944893e-1
1/n^(7/10) = 1.22822802611579e-1
difference = -5.22201633331434e-3
n = 50 , left side = 5.54458428059442e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 9.22685785179153e-3
n = 100 , left side = 2.77258872001896e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 1.20848298551601e-2
n = 200 , left side = 1.38629436111991e-2
1/n^(7/10) = 2.45063709469745e-2
difference = 1.06434273357754e-2
n = 500 , left side = 5.54517744447935e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 7.35872279848497e-3

```


$$x^2$$

```

n = 10 , left side = 2.44068957215094e-1
          1/n^(7/10) = 1.99526231496888e-1
          difference = -4.45427257182058e-2
n = 20 , left side = 1.73393496362237e-1
          1/n^(7/10) = 1.22822802611579e-1
          difference = -5.05706937506581e-2
n = 50 , left side = 6.39143140576404e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 7.58386600095390e-4
n = 100 , left side = 2.98438925970291e-2
          1/n^(7/10) = 3.98107170553497e-2
          difference = 9.96682445832066e-3
n = 200 , left side = 1.43924449634693e-2
          1/n^(7/10) = 2.45063709469745e-2
          difference = 1.01139259835052e-2
n = 500 , left side = 5.62989766084276e-3
          1/n^(7/10) = 1.29039002429643e-2
          difference = 7.27400258212156e-3

```

$$x^3$$

```

n = 10 , left side = 2.70532081486956e-1
          1/n^(7/10) = 1.99526231496888e-1
          difference = -7.10058499900676e-2
n = 20 , left side = 1.75303858260733e-1
          1/n^(7/10) = 1.22822802611579e-1
          difference = -5.24810556491540e-2
n = 50 , left side = 5.53534166881951e-2
          1/n^(7/10) = 6.46727006577358e-2
          difference = 9.31928396954068e-3
n = 100 , left side = 2.41049670707777e-2
          1/n^(7/10) = 3.98107170553497e-2
          difference = 1.57057499845720e-2
n = 200 , left side = 1.12081526845386e-2

```

```

1/n^(7/10) = 2.45063709469745e-2
difference = 1.32982182624359e-2
n = 500 , left side = 4.28703175655951e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 8.61686848640482e-3

```

$$x^4$$

```

n = 10 , left side = 2.71811655321145e-1
1/n^(7/10) = 1.99526231496888e-1
difference = -7.22854238242565e-2
n = 20 , left side = 1.59700780897926e-1
1/n^(7/10) = 1.22822802611579e-1
difference = -3.68779782863466e-2
n = 50 , left side = 4.27867893894554e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 2.18859112682804e-2
n = 100 , left side = 1.73213915349858e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 2.24893255203640e-2
n = 200 , left side = 7.76000677274204e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.67463641742325e-2
n = 500 , left side = 2.90182870840223e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.00020715345621e-2

```

$$x^{10}$$

```

n = 10 , left side = 1.86850823331552e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 1.26754081653362e-2
n = 20 , left side = 6.58347789205075e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 5.69880236910715e-2
n = 50 , left side = 4.87085072187464e-3

```

```

1/n^(7/10) = 6.46727006577358e-2
difference = 5.98018499358611e-2
n = 100 , left side = 1.09970741430996e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.87110096410398e-2
n = 200 , left side = 3.82804893814586e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.41235660531599e-2
n = 500 , left side = 1.24276968148264e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.27796232748161e-2

```

x0 = 1/2, Power = 7/10, lamda = 1/4, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.96080052578450e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.79918226239043e-1
n = 20 , left side = 2.08299179386359e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.01992884672943e-1
n = 50 , left side = 1.26071386109592e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 5.20655620467766e-2
n = 100 , left side = 6.81015763376169e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.30005594215880e-2
n = 200 , left side = 3.53432821873112e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.09720427282434e-2
n = 500 , left side = 1.44549650366488e-3
1/n^(7/10) = 1.29039002429643e-2

```

difference = 1.14584037392994e-2

x

n = 10 , left side = 5.68097172543791e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.42716514242509e-1
n = 20 , left side = 6.32752819000137e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 5.95475207115653e-2
n = 50 , left side = 2.77227554082915e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 3.69499452494443e-2
n = 100 , left side = 1.38629435997853e-2
1/n^(7/10) = 3.98107170553497e-2
difference = 2.59477734555644e-2
n = 200 , left side = 6.93147180559972e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 1.75748991413748e-2
n = 500 , left side = 2.77258872223995e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.01313115207244e-2

x^2

n = 10 , left side = 1.47872752410878e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 5.16534790860099e-2
n = 20 , left side = 9.78605569036922e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 2.49622457078869e-2
n = 50 , left side = 3.38868135786705e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 3.07858870790653e-2
n = 100 , left side = 1.54044053860309e-2
1/n^(7/10) = 3.98107170553497e-2

```

        difference = 2.44063116693188e-2
n = 200 , left side = 7.31683725369431e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 1.71895336932802e-2
n = 500 , left side = 2.83424719393494e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.00696530490294e-2

```

$$x^3$$

```

n = 10 , left side = 1.73192000431426e-1
        1/n^(7/10) = 1.99526231496888e-1
        difference = 2.63342310654625e-2
n = 20 , left side = 1.03864925521532e-1
        1/n^(7/10) = 1.22822802611579e-1
        difference = 1.89578770900470e-2
n = 50 , left side = 3.05073733511566e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 3.41653273065792e-2
n = 100 , left side = 1.27681795752464e-2
        1/n^(7/10) = 3.98107170553497e-2
        difference = 2.70425374801034e-2
n = 200 , left side = 5.78399942625288e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 1.87223715207216e-2
n = 500 , left side = 2.17239948281706e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.07315007601473e-2

```

$$x^4$$

```

n = 10 , left side = 1.80257354696213e-1
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.92688768006753e-2
n = 20 , left side = 9.63498090064417e-2
        1/n^(7/10) = 1.22822802611579e-1

```

```

        difference = 2.64729936051373e-2
n = 50 , left side = 2.41914336230855e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 4.04812670346503e-2
n = 100 , left side = 9.37035404303910e-3
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.04403630123106e-2
n = 200 , left side = 4.05904957310241e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.04473213738721e-2
n = 500 , left side = 1.47973714572865e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.14241630972357e-2

```

$$x^{10}$$

```

n = 10 , left side = 1.19260700953333e-1
        1/n^(7/10) = 1.99526231496888e-1
        difference = 8.02655305435547e-2
n = 20 , left side = 3.73465208237525e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 8.54762817878265e-2
n = 50 , left side = 2.80162956157206e-3
        1/n^(7/10) = 6.46727006577358e-2
        difference = 6.18710710961637e-2
n = 100 , left side = 6.32905221145845e-4
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.91778118342039e-2
n = 200 , left side = 2.12043953336508e-4
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.42943269936380e-2
n = 500 , left side = 6.54808182457762e-5
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.28384194247185e-2

```

x0 = 1/2, Power = 7/10, lamda = 1/4, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 8.10134985303516e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.18512732966536e-1
n = 20 , left side = 1.68560763315217e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.05966726280057e-1
n = 50 , left side = 1.97802106438832e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.26946795933474e-2
n = 100 , left side = 4.80026592954430e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.93306904623953e-2
n = 200 , left side = 1.19239646471425e-4
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.43871313005031e-2
n = 500 , left side = 1.90449720897101e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28848552708746e-2

```

$$x$$

```

n = 10 , left side = 4.91637605442458e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.50362470952642e-1
n = 20 , left side = 3.39394250680042e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.19428860104779e-1
n = 50 , left side = 1.64890433890230e-6
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.46710517533969e-2

```

```

n = 100 , left side = 5.86114490275236e-12
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.98107170494886e-2
n = 200 , left side = 0.000000000000000e0
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.45063709469745e-2
n = 500 , left side = 0.000000000000000e0
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.29039002429643e-2

```

$$x^2$$

```

n = 10 , left side = 5.25230782287648e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.47003153268123e-1
n = 20 , left side = 2.83373415030339e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 9.44854611085451e-2
n = 50 , left side = 5.39457664221732e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.92781240155184e-2
n = 100 , left side = 1.34928057788158e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.84614364774681e-2
n = 200 , left side = 3.37320146703013e-4
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.41690508002715e-2
n = 500 , left side = 5.39712234724488e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28499290194919e-2

```

$$x^3$$

```

n = 10 , left side = 8.22234506283697e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.17302780868518e-1

```


$n = 20$, left side = 4.36057626242193e-2
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = 7.92170399873598e-2
 $n = 50$, left side = 8.09259320656272e-3
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = 5.65801074511730e-2
 $n = 100$, left side = 2.02392086959333e-3
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = 3.77867961857564e-2
 $n = 200$, left side = 5.05980220054492e-4
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = 2.40003907269200e-2
 $n = 500$, left side = 8.09568352087009e-5
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = 1.28229434077556e-2

$$x^4$$

$n = 10$, left side = 1.01052973293562e-1
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = 9.84732582033261e-2
 $n = 20$, left side = 4.67481413489399e-2
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = 7.60746612626391e-2
 $n = 50$, left side = 8.21282675171919e-3
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = 5.64598739060166e-2
 $n = 100$, left side = 2.03145927015694e-3
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = 3.77792577851928e-2
 $n = 200$, left side = 5.06451370153385e-4
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = 2.39999195768211e-2
 $n = 500$, left side = 8.09688966512678e-5
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = 1.28229313463131e-2

$$x^{10}$$

```

n = 10 , left side = 7.01207124827065e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.29405519014181e-1
n = 20 , left side = 2.01574704246336e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.02665332186945e-1
n = 50 , left side = 1.42480124701184e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.32478994107239e-2
n = 100 , left side = 2.63178146244861e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.95475389091049e-2
n = 200 , left side = 6.08599933529959e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44455109536215e-2
n = 500 , left side = 9.52678518000342e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28943734577843e-2

```

x0 = 1/2, Power = 7/10, lamda = 1/2, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 5.33369660315088e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.46189265465379e-1
n = 20 , left side = 3.23728371506258e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 9.04499654609532e-2
n = 50 , left side = 1.39366391997124e-2
          1/n^(7/10) = 6.46727006577358e-2

```

```

        difference = 5.07360614580233e-2
n = 100 , left side = 7.14621856758857e-3
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.26644984877612e-2
n = 200 , left side = 3.61961230894126e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.08867586380332e-2
n = 500 , left side = 1.45930076253686e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.14445994804275e-2

```

x

```

n = 10 , left side = 1.29071817455777e-1
        1/n^(7/10) = 1.99526231496888e-1
        difference = 7.04544140411110e-2
n = 20 , left side = 6.92514035886481e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 5.35713990229309e-2
n = 50 , left side = 2.77258876437841e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 3.69468130139516e-2
n = 100 , left side = 1.38629438314646e-2
        1/n^(7/10) = 3.98107170553497e-2
        difference = 2.59477732238852e-2
n = 200 , left side = 6.93147191573251e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 1.75748990312420e-2
n = 500 , left side = 2.77258876629327e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.01313114766711e-2

```

x^2

```

n = 10 , left side = 1.77390913518807e-1
        1/n^(7/10) = 1.99526231496888e-1

```

```

        difference = 2.21353179780806e-2
n = 20 , left side = 8.30737213816240e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 3.97490812299550e-2
n = 50 , left side = 2.99438930027937e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 3.47288076549421e-2
n = 100 , left side = 1.44174451738843e-2
        1/n^(7/10) = 3.98107170553497e-2
        difference = 2.53932718814654e-2
n = 200 , left side = 7.07009725133750e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 1.74362736956370e-2
n = 500 , left side = 2.79476881998975e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.01091314229746e-2

```

$$x^3$$

```

n = 10 , left side = 1.81706840017598e-1
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.78193914792899e-2
n = 20 , left side = 7.48642399521749e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 4.79585626594041e-2
n = 50 , left side = 2.42632851043290e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 4.04094155534067e-2
n = 100 , left side = 1.12466925545658e-2
        1/n^(7/10) = 3.98107170553497e-2
        difference = 2.85640245007839e-2
n = 200 , left side = 5.40875852362388e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 1.90976124233506e-2
n = 500 , left side = 2.11285351660367e-3
        1/n^(7/10) = 1.29039002429643e-2

```

difference = 1.07910467263606e-2

$$x^4$$

n = 10 , left side = 1.67787178069694e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 3.17390534271940e-2
n = 20 , left side = 6.03397370785075e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 6.24830655330716e-2
n = 50 , left side = 1.74893080184918e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 4.71833926392440e-2
n = 100 , left side = 7.79966635748697e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.20110506978628e-2
n = 200 , left side = 3.67816819644864e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.08282027505259e-2
n = 500 , left side = 1.41984974971847e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.14840504932459e-2

$$x^{10}$$

n = 10 , left side = 7.68027797084298e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.22723451788458e-1
n = 20 , left side = 9.76146773173466e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.13061334879844e-1
n = 50 , left side = 1.13068528957820e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 6.35420153681576e-2
n = 100 , left side = 3.88474877995726e-4
1/n^(7/10) = 3.98107170553497e-2

```

difference = 3.94222421773540e-2
n = 200 , left side = 1.62038602246220e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.43443323447283e-2
n = 500 , left side = 5.81892063837631e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28457110365806e-2

```

x0 = 1/2, Power = 7/10, lamda = 1/2, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.96445247579275e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.79881706738961e-1
n = 20 , left side = 1.48888789279412e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.07933923683638e-1
n = 50 , left side = 6.77605354912070e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.78966471086151e-2
n = 100 , left side = 3.52568356510685e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.62850334902429e-2
n = 200 , left side = 1.79801315131500e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.27083577956595e-2
n = 500 , left side = 7.27768804897111e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.21761314380672e-2

```

$$x$$

$n = 10$, left side = 6.38935662496244e-2
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = 1.35632665247264e-1
 $n = 20$, left side = 3.46235470985199e-2
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = 8.81992555130592e-2
 $n = 50$, left side = 1.38629429714340e-2
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = 5.08097576863018e-2
 $n = 100$, left side = 6.93147149063511e-3
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = 3.28792455647146e-2
 $n = 200$, left side = 3.46573574531772e-3
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = 2.10406352016568e-2
 $n = 500$, left side = 1.38629429812720e-3
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = 1.15176059448371e-2

$$x^2$$

$n = 10$, left side = 1.01231195168683e-1
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = 9.82950363282052e-2
 $n = 20$, left side = 4.48623512045705e-2
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = 7.79604514070085e-2
 $n = 50$, left side = 1.55044047114581e-2
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = 4.91682959462776e-2
 $n = 100$, left side = 7.34183692697737e-3
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = 3.24688801283724e-2
 $n = 200$, left side = 3.56832710440319e-3
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = 2.09380438425713e-2

n = 500 , left side = 1.40270891558075e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.15011913273836e-2

$$x^3$$

n = 10 , left side = 1.09051180141690e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 9.04750513551982e-2
n = 20 , left side = 4.22970110547811e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 8.05257915567980e-2
n = 50 , left side = 1.29223379237350e-2
1/n^(7/10) = 6.46727006577358e-2
difference = 5.17503627340007e-2
n = 100 , left side = 5.82201903348833e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.39886980218614e-2
n = 200 , left side = 2.75417225524147e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.17521986917330e-2
n = 500 , left side = 1.06440558786389e-3
1/n^(7/10) = 1.29039002429643e-2
difference = 1.18394946551004e-2

$$x^4$$

n = 10 , left side = 1.02540884681400e-1
1/n^(7/10) = 1.99526231496888e-1
difference = 9.69853468154883e-2
n = 20 , left side = 3.49981874860216e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 8.78246151255575e-2
n = 50 , left side = 9.52960631552958e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.51430943422062e-2


```

n = 100 , left side = 4.09764755001929e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.57130695053304e-2
n = 200 , left side = 1.88876104704880e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.26176098999257e-2
n = 500 , left side = 7.17895958025183e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.21860042849391e-2

```

$$x^{10}$$

```

n = 10 , left side = 4.39432749437958e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.55582956553092e-1
n = 20 , left side = 5.59881198565948e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.17223990625920e-1
n = 50 , left side = 6.58122885628435e-4
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.40145777721073e-2
n = 100 , left side = 2.17135216419696e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.95935818389300e-2
n = 200 , left side = 8.67799400496487e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44195910069249e-2
n = 500 , left side = 3.00238003038830e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28738764426604e-2

```

x0 = 1/2, Power = 7/10, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.88583358245119e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.80667895672376e-1
n = 20 , left side = 3.43036193768762e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.19392440673891e-1
n = 50 , left side = 5.15745217625541e-4
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.41569554401102e-2
n = 100 , left side = 1.28085535782807e-4
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.96826315195669e-2
n = 200 , left side = 3.19697272262998e-5
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.44744012197482e-2
n = 500 , left side = 5.11285838666886e-6
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28987873845777e-2

```

$$x$$

```

n = 10 , left side = 3.03097878388953e-3
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.96495252712998e-1
n = 20 , left side = 1.79813965206499e-5
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.22804821215058e-1
n = 50 , left side = 5.04973840520506e-12
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.46727006526860e-2
n = 100 , left side = 1.11022302462516e-16
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.98107170553496e-2
n = 200 , left side = 0.000000000000000e0

```

```

1/n^(7/10) = 2.45063709469745e-2
difference = 2.45063709469745e-2
n = 500 , left side = 5.55111512312578e-17
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29039002429643e-2

```

$$x^2$$

```

n = 10 , left side = 3.13288100022530e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.68197421494635e-1
n = 20 , left side = 9.02981956494553e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.13792983046634e-1
n = 50 , left side = 1.44928066358080e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 6.32234199941550e-2
n = 100 , left side = 3.62320167826280e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.94483968875235e-2
n = 200 , left side = 9.05800419565561e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44157909050179e-2
n = 500 , left side = 1.44928067130623e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28894074362513e-2

```

$$x^3$$

```

n = 10 , left side = 4.78600213916086e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.51666210105279e-1
n = 20 , left side = 1.35520533206201e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.09270749290959e-1
n = 50 , left side = 2.17392099772531e-3

```

```

1/n^(7/10) = 6.46727006577358e-2
difference = 6.24987796600104e-2
n = 100 , left side = 5.43480251739309e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.92672368036104e-2
n = 200 , left side = 1.35870062934806e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.43705008840397e-2
n = 500 , left side = 2.17392100695102e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28821610328948e-2

```

$$x^4$$

```

n = 10 , left side = 5.14533906505907e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.48072840846297e-1
n = 20 , left side = 1.38732244570932e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.08949578154486e-1
n = 50 , left side = 2.18227896612364e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 6.24904216916121e-2
n = 100 , left side = 5.44002624822695e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.92667144305270e-2
n = 200 , left side = 1.35902711252578e-4
1/n^(7/10) = 2.45063709469745e-2
difference = 2.43704682357219e-2
n = 500 , left side = 2.17400458664835e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28821601970978e-2

```

$$x^{10}$$

```

n = 10 , left side = 2.39574413183785e-2

```

```

1/n^(7/10) = 1.99526231496888e-1
difference = 1.75568790178510e-1
n = 20 , left side = 2.97086004062642e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.19851942570953e-1
n = 50 , left side = 2.83599622557975e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.43891010351778e-2
n = 100 , left side = 6.54249762594573e-5
1/n^(7/10) = 3.98107170553497e-2
difference = 3.97452920790903e-2
n = 200 , left side = 1.60297404174447e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44903412065571e-2
n = 500 , left side = 2.55030753047421e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.29013499354338e-2

```

x0 = 1/2, Power = 7/10, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 3.15884348850566e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.67937796611831e-1
n = 20 , left side = 1.69988381319672e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.05823964479612e-1
n = 50 , left side = 7.10929624048973e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.75634044172460e-2
n = 100 , left side = 3.60940309324842e-3
1/n^(7/10) = 3.98107170553497e-2

```

```

        difference = 3.62013139621013e-2
n = 200 , left side = 1.81874048780928e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.26876304591652e-2
n = 500 , left side = 7.30913664388266e-4
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.21729865785761e-2

```

x

```

n = 10 , left side = 6.92279510900182e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.30298280406870e-1
n = 20 , left side = 3.46421335720477e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 8.81806690395313e-2
n = 50 , left side = 1.38568544082612e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 5.08158462494745e-2
n = 100 , left side = 6.92842720413067e-3
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.28822898512191e-2
n = 200 , left side = 3.46421360206539e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.10421573449091e-2
n = 500 , left side = 1.38568544082618e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.15182148021381e-2

```

x^2

```

n = 10 , left side = 8.55451218222509e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.13981109674637e-1
n = 20 , left side = 3.87308178004870e-2
        1/n^(7/10) = 1.22822802611579e-1

```

```

        difference = 8.40919848110921e-2
n = 50 , left side = 1.45110441151024e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 5.01616565426334e-2
n = 100 , left side = 7.09197463084094e-3
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.27187424245088e-2
n = 200 , left side = 3.50510045874292e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.10012704882316e-2
n = 500 , left side = 1.39222733789462e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.15116729050697e-2

```

$$x^3$$

```

n = 10 , left side = 7.91097899306038e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.20416441566284e-1
n = 20 , left side = 3.24567771921774e-2
        1/n^(7/10) = 1.22822802611579e-1
        difference = 9.03660254194017e-2
n = 50 , left side = 1.13958230614945e-2
        1/n^(7/10) = 6.46727006577358e-2
        difference = 5.32768775962413e-2
n = 100 , left side = 5.44437875504283e-3
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.43663383003069e-2
n = 200 , left side = 2.65983263805031e-3
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.18465383089242e-2
n = 500 , left side = 1.04909882391729e-3
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.18548014190470e-2

```

$$x^4$$

$n = 10$, left side = $6.53260321168637e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.34200199380024e-1$
 $n = 20$, left side = $2.41899798353885e-2$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $9.86328227761906e-2$
 $n = 50$, left side = $7.95482775153313e-3$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $5.67178729062026e-2$
 $n = 100$, left side = $3.71509170320668e-3$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.60956253521431e-2$
 $n = 200$, left side = $1.79412654760111e-3$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.27122443993734e-2$
 $n = 500$, left side = $7.02699493465395e-4$
 $1/n^{(7/10)} = 1.29039002429643e-2$
difference = $1.22012007494989e-2$

$$x^{10}$$

$n = 10$, left side = $1.23005431860469e-2$
 $1/n^{(7/10)} = 1.99526231496888e-1$
difference = $1.87225688310841e-1$
 $n = 20$, left side = $1.95443229260425e-3$
 $1/n^{(7/10)} = 1.22822802611579e-1$
difference = $1.20868370318975e-1$
 $n = 50$, left side = $4.11078488591595e-4$
 $1/n^{(7/10)} = 6.46727006577358e-2$
difference = $6.42616221691442e-2$
 $n = 100$, left side = $1.66923378278077e-4$
 $1/n^{(7/10)} = 3.98107170553497e-2$
difference = $3.96437936770717e-2$
 $n = 200$, left side = $7.51857729315232e-5$
 $1/n^{(7/10)} = 2.45063709469745e-2$
difference = $2.44311851740430e-2$


```

n = 500 , left side = 2.82350791906388e-5
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.28756651637737e-2

```

x0 = 1/2, Power = 7/10, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 1.40292664147630e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.85496965082125e-1
n = 20 , left side = 8.09536658966781e-3
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.14727436021911e-1
n = 50 , left side = 3.49396872417307e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.11787319335627e-2
n = 100 , left side = 1.79071372384143e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.80200033315083e-2
n = 200 , left side = 9.06435452010634e-4
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.35999354949639e-2
n = 500 , left side = 3.65254093297129e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.25386461496672e-2

```

$$x$$

```

n = 10 , left side = 3.46541098802871e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.64872121616601e-1
n = 20 , left side = 1.73420259977516e-2

```

```

1/n^(7/10) = 1.22822802611579e-1
difference = 1.05480776613827e-1
n = 50 , left side = 6.93681090501508e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.77358897527207e-2
n = 100 , left side = 3.46840545250782e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.63423116028419e-2
n = 200 , left side = 1.73420272625380e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.27721682207207e-2
n = 500 , left side = 6.93681090501386e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.22102191524629e-2

```

$$x^2$$

```

n = 10 , left side = 4.73903023658121e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.52135929131076e-1
n = 20 , left side = 2.05308163937205e-2
1/n^(7/10) = 1.22822802611579e-1
difference = 1.02291986217859e-1
n = 50 , left side = 7.44701748385268e-3
1/n^(7/10) = 6.46727006577358e-2
difference = 5.72256831738831e-2
n = 100 , left side = 3.59595709721700e-3
1/n^(7/10) = 3.98107170553497e-2
difference = 3.62147599581327e-2
n = 200 , left side = 1.76609063743116e-3
1/n^(7/10) = 2.45063709469745e-2
difference = 2.27402803095433e-2
n = 500 , left side = 6.98783156289906e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.22051170866744e-2

```

$$x^3$$

```

n = 10 , left side = 4.63238425467439e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.53202388950144e-1
n = 20 , left side = 1.79448446203553e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.04877957991224e-1
n = 50 , left side = 5.97784700290624e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 5.86948536548295e-2
n = 100 , left side = 2.79387267593101e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.70168443794187e-2
n = 200 , left side = 1.34863905139215e-3
          1/n^(7/10) = 2.45063709469745e-2
          difference = 2.31577318955823e-2
n = 500 , left side = 5.27923845514616e-4
          1/n^(7/10) = 1.29039002429643e-2
          difference = 1.23759763974497e-2

```

$$x^4$$

```

n = 10 , left side = 3.94349040023764e-2
          1/n^(7/10) = 1.99526231496888e-1
          difference = 1.60091327494512e-1
n = 20 , left side = 1.37990548305422e-2
          1/n^(7/10) = 1.22822802611579e-1
          difference = 1.09023747781037e-1
n = 50 , left side = 4.25445841424152e-3
          1/n^(7/10) = 6.46727006577358e-2
          difference = 6.04182422434942e-2
n = 100 , left side = 1.92806775614625e-3
          1/n^(7/10) = 3.98107170553497e-2
          difference = 3.78826492992035e-2
n = 200 , left side = 9.15246967505515e-4

```

```

1/n^(7/10) = 2.45063709469745e-2
difference = 2.35911239794690e-2
n = 500 , left side = 3.54513590363287e-4
1/n^(7/10) = 1.29039002429643e-2
difference = 1.25493866526010e-2

```

$$x^{10}$$

```

n = 10 , left side = 7.27638680160743e-3
1/n^(7/10) = 1.99526231496888e-1
difference = 1.92249844695281e-1
n = 20 , left side = 1.19062238971356e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.21632180221865e-1
n = 50 , left side = 2.37641399649618e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.44350592580861e-2
n = 100 , left side = 9.15159839724954e-5
1/n^(7/10) = 3.98107170553497e-2
difference = 3.97192010713772e-2
n = 200 , left side = 3.96334550487991e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44667374919257e-2
n = 500 , left side = 1.44549073568421e-5
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28894453356075e-2

```

```

-----
x0 = 1/2, Power = 7/10, lamda = 1,  q = 1
-----

```

$$x^{\frac{1}{3}}$$

```

n = 10 , left side = 4.43886557706544e-3
1/n^(7/10) = 1.99526231496888e-1

```

```

        difference = 1.95087365919823e-1
n = 20 , left side = 1.03630053646838e-3
        1/n^(7/10) = 1.22822802611579e-1
        difference = 1.21786502075111e-1
n = 50 , left side = 1.63624778015015e-4
        1/n^(7/10) = 6.46727006577358e-2
        difference = 6.45090758797207e-2
n = 100 , left side = 4.08320646757554e-5
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.97698849906740e-2
n = 200 , left side = 1.02034164284337e-5
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.44961675305461e-2
n = 500 , left side = 1.63234093675246e-6
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.29022679020276e-2

```

x

```

n = 10 , left side = 1.58663982219309e-5
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.99510365098666e-1
n = 20 , left side = 6.52684128965575e-10
        1/n^(7/10) = 1.22822802611579e-1
        difference = 1.22822801958895e-1
n = 50 , left side = 0.000000000000000e0
        1/n^(7/10) = 6.46727006577358e-2
        difference = 6.46727006577358e-2
n = 100 , left side = 0.000000000000000e0
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.98107170553497e-2
n = 200 , left side = 0.000000000000000e0
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.45063709469745e-2
n = 500 , left side = 0.000000000000000e0
        1/n^(7/10) = 1.29039002429643e-2

```

difference = 1.29039002429643e-2

$$x^2$$

n = 10 , left side = 1.15428308402608e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.87983400656627e-1
n = 20 , left side = 2.89205247195795e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.19930750139621e-1
n = 50 , left side = 4.62728557739434e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.42099720999963e-2
n = 100 , left side = 1.15682139434858e-4
1/n^(7/10) = 3.98107170553497e-2
difference = 3.96950349159149e-2
n = 200 , left side = 2.89205348588673e-5
1/n^(7/10) = 2.45063709469745e-2
difference = 2.44774504121156e-2
n = 500 , left side = 4.62728557748537e-6
1/n^(7/10) = 1.29039002429643e-2
difference = 1.28992729573868e-2

$$x^3$$

n = 10 , left side = 1.73200399632407e-2
1/n^(7/10) = 1.99526231496888e-1
difference = 1.82206191533647e-1
n = 20 , left side = 4.33807899343377e-3
1/n^(7/10) = 1.22822802611579e-1
difference = 1.18484723618145e-1
n = 50 , left side = 6.94092836609345e-4
1/n^(7/10) = 6.46727006577358e-2
difference = 6.39786078211264e-2
n = 100 , left side = 1.73523209152371e-4
1/n^(7/10) = 3.98107170553497e-2

```

        difference = 3.96371938461974e-2
n = 200 , left side = 4.33808022881066e-5
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.44629901446864e-2
n = 500 , left side = 6.94092836614479e-6
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.28969593145982e-2

```

$$x^4$$

```

n = 10 , left side = 1.77823913197880e-2
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.81743840177100e-1
n = 20 , left side = 4.36734781351719e-3
        1/n^(7/10) = 1.22822802611579e-1
        difference = 1.18455454798062e-1
n = 50 , left side = 6.94842122375819e-4
        1/n^(7/10) = 6.46727006577358e-2
        difference = 6.39778585353599e-2
n = 100 , left side = 1.73570039512738e-4
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.96371470158370e-2
n = 200 , left side = 4.33837291856087e-5
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.44629872177889e-2
n = 500 , left side = 6.94100329461100e-6
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.28969592396697e-2

```

$$x^{10}$$

```

n = 10 , left side = 4.06239089134456e-3
        1/n^(7/10) = 1.99526231496888e-1
        difference = 1.95463840605543e-1
n = 20 , left side = 6.12650024829540e-4
        1/n^(7/10) = 1.22822802611579e-1

```

```

        difference = 1.22210152586750e-1
n = 50 , left side = 8.38305099970637e-5
        1/n^(7/10) = 6.46727006577358e-2
        difference = 6.45888701477387e-2
n = 100 , left side = 2.04889255338266e-5
        1/n^(7/10) = 3.98107170553497e-2
        difference = 3.97902281298159e-2
n = 200 , left side = 5.09329964822080e-6
        1/n^(7/10) = 2.45063709469745e-2
        difference = 2.45012776473263e-2
n = 500 , left side = 8.13635935028711e-7
        1/n^(7/10) = 1.29039002429643e-2
        difference = 1.29030866070293e-2

```


4 Real-valued neural network approximation based on the q-deformed and λ -parametrized Hyperbolic Tangent - part 2

```
[ ]: RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1]    #deformation parameter lamda over (0, 1] - these are the beta values in the formula
qs = [1/4, 1/2, 1]    #deformation coefficient

funcs = [sin(x), cos(x)]    #choice of functions
a = -pi    #the interval
b = pi    #the interval
x0s= [pi/4, pi/2, 3*pi/4]

#####
for x0 in x0s:
    #####
    for power in powers:    #going over various powers for 1/
        ↪ n^power

        ↪
        ↪#####
        for lamda in lamdas:    #going over each lamda value

            ↪
            ↪#####
            for q in qs:    #going over each q value

                ↪
                ↪#####
                print()
                print()

                ↪
                ↪print("-----")
                print("x0 = " + str(x0)+", Power = "+
                ↪str(power)+ ", lamda = "+ str(lamda) + ", q = " + str(q))
```

```

    print("-----")

    #the activation function
    phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
    (e^(lamda*x)+q*e^(-lamda*x))    #formula 18.1

    #
    G(x) = 1/4*(phi(x+1) - phi(x-1))    #formula 18.9

    #####
    for i in range(len(funcs)):
        #####
        f(x)=funcs[i]
        show(f(x))
        for n in [2000, 5000]:
            #def L(n, f, x):    #real-valued
            #    return sum(f(k/n)*G(n*x-k) for k
            #in [ceil(n*a),...,floor(n*b)])/sum(G(n*x-k) for k
            #in [ceil(n*a),...,floor(n*b)])
            #leftSide = abs(L(n,f,x0)-f(x0))
            leftSide = abs(sum(f(k/n)*G(n*x0-k)
            for k in [ceil(n*a),...,floor(n*b)])/sum(G(n*x0-k) for k
            in [ceil(n*a),...,floor(n*b)])-f(x0))
            val1 = n
            val2 = leftSide.n()
            val3 = 1/(n^power).n()
            print("n = "+str(val1), "1/
            left side = "+str(val2), "\n
            n^("+str(power)+") = "+str(val3), "\n
            difference = "+str(val3-val2))

```

x0 = 1/4*pi, Power = 3/10, lamda = 1/4, q = 1/4

$\sin(x)$

n = 2000 , left side = 9.78384108846653e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.01278134147511e-1
n = 5000 , left side = 3.91803600351293e-4
1/n^(3/10) = 7.76799609715734e-2
difference = 7.72881573712221e-2

$\cos(x)$

n = 2000 , left side = 9.82128243529390e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.01274390012828e-1
n = 5000 , left side = 3.92402662610758e-4
1/n^(3/10) = 7.76799609715734e-2
difference = 7.72875583089626e-2

x0 = 1/4*pi, Power = 3/10, lamda = 1/4, q = 1/2

$\sin(x)$

n = 2000 , left side = 4.88765734909080e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.01767752521448e-1
n = 5000 , left side = 1.95833577701698e-4
1/n^(3/10) = 7.76799609715734e-2
difference = 7.74841273938717e-2

$$\cos(x)$$

```

n = 2000 , left side = 4.91490676762352e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.01765027579595e-1
n = 5000 , left side = 1.96269568849661e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.74836914027237e-2

```

$x_0 = 1/4\pi$, Power = 3/10, lamda = 1/4, q = 1

$$\sin(x)$$

```

n = 2000 , left side = 1.19260542763566e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02255325650930e-1
n = 5000 , left side = 1.90817055045756e-7
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76797701545183e-2

```

$$\cos(x)$$

```

n = 2000 , left side = 1.19260542796873e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02255325650929e-1
n = 5000 , left side = 1.90817054490644e-7
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76797701545189e-2

```

$x_0 = 1/4\pi$, Power = 3/10, lamda = 1/2, q = 1/4

$\sin(x)$

n = 2000 , left side = 4.89638706925866e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.01766879549431e-1
n = 5000 , left side = 1.95973196964960e-4
1/n^(3/10) = 7.76799609715734e-2
difference = 7.74839877746084e-2

$\cos(x)$

n = 2000 , left side = 4.90618935763076e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.01765899320594e-1
n = 5000 , left side = 1.96130033619624e-4
1/n^(3/10) = 7.76799609715734e-2
difference = 7.74838309379538e-2

x0 = 1/4*pi, Power = 3/10, lamda = 1/2, q = 1/2

$\sin(x)$

n = 2000 , left side = 2.44701705652384e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02011816550705e-1
n = 5000 , left side = 9.79677681450797e-5
1/n^(3/10) = 7.76799609715734e-2
difference = 7.75819932034283e-2

$\cos(x)$

n = 2000 , left side = 2.45427135862286e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02011091120495e-1

```

n = 5000 , left side = 9.80838370118198e-5
1/n^(3/10) = 7.76799609715734e-2
difference = 7.75818771345616e-2

```

```

x0 = 1/4*pi, Power = 3/10, lamda = 1/2, q = 1

```

$\sin(x)$

```

n = 2000 , left side = 3.20260086117408e-7
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02256197996271e-1
n = 5000 , left side = 5.12400810670499e-8
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76799097314923e-2

```

$\cos(x)$

```

n = 2000 , left side = 3.20237314221927e-7
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02256198019043e-1
n = 5000 , left side = 5.12395328389204e-8
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76799097320405e-2

```

```

x0 = 1/4*pi, Power = 3/10, lamda = 1, q = 1/4

```

$\sin(x)$

```

n = 2000 , left side = 2.44927550096308e-4
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02011590706261e-1

```

```

n = 5000 , left side = 9.79606247285547e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.75820003468448e-2

```

$\cos(x)$

```

n = 2000 , left side = 2.45216626713507e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02011301629644e-1
n = 5000 , left side = 9.80068816603952e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.75819540899130e-2

```

```

-----
x0 = 1/4*pi, Power = 3/10, lamda = 1,  q = 1/2
-----

```

$\sin(x)$

```

n = 2000 , left side = 1.22509259644299e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02134008996713e-1
n = 5000 , left side = 4.90340684149437e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76309269031584e-2

```

$\cos(x)$

```

n = 2000 , left side = 1.22734955049442e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02133783301308e-1
n = 5000 , left side = 4.90701470647892e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76308908245086e-2

```

x0 = 1/4*pi, Power = 3/10, lamda = 1, q = 1

$\sin(x)$

n = 2000 , left side = 2.12255174658615e-7
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02256306001183e-1
n = 5000 , left side = 1.90105314823441e-8
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76799419610419e-2

$\cos(x)$

n = 2000 , left side = 7.88483400793183e-9
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02256510371523e-1
n = 5000 , left side = 1.37092714957987e-8
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76799472623019e-2

x0 = 1/4*pi, Power = 1/2, lamda = 1/4, q = 1/4

$\sin(x)$

n = 2000 , left side = 9.78384108846653e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.13822956661512e-2
n = 5000 , left side = 3.91803600351293e-4
1/n^(1/2) = 1.41421356237310e-2
difference = 1.37503320233797e-2

$$\cos(x)$$

```

n = 2000 , left side = 9.82128243529390e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.13785515314685e-2
n = 5000 , left side = 3.92402662610758e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.37497329611202e-2

```

 $x_0 = 1/4\pi$, Power = 1/2, lamda = 1/4, q = 1/2

$$\sin(x)$$

```

n = 2000 , left side = 4.88765734909080e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.18719140400888e-2
n = 5000 , left side = 1.95833577701698e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.39463020460293e-2

```

$$\cos(x)$$

```

n = 2000 , left side = 4.91490676762352e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.18691890982355e-2
n = 5000 , left side = 1.96269568849661e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.39458660548813e-2

```

```

[1]: RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1]    #deformation parameter lamda over (0,
    ↪ 1] - these are the beta values in the formula
qs = [1/4, 1/2, 1]    #deformation coefficient

funcs = [x^(1/3), x, x^2, x^3, x^4, x^10]    #choice of
    ↪ functions
a = -1    #the interval
b = 1    #the interval
x0=1/2

for power in powers:
    ↪
    ↪#####
    for lamda in lamdas:    #going over each lamda value
        ↪
        ↪#####
        for q in qs:        #going over each q value
            ↪
            ↪#####
            print()
            print()
            ↪
            ↪print("-----")
            print("x0 = " + str(x0)+", Power = "+ str(power)+
            ↪", lamda = "+ str(lamda) + ", q = " + str(q))
            ↪
            ↪print("-----")

            #the activation function
            phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
            ↪(e^(lamda*x)+q*e^(-lamda*x))    #formula 18.1

```

```

#
G(x) = 1/4*(phi(x+1) - phi(x-1))      #formula 18.9

└
↪#####
    for i in range(len(funcs)):
└
↪#####
        f(x)=funcs[i]
        show(f(x))
        for n in [2000, 5000]:
            #def L(n, f, x):      #real-valued linear
↪neural network operators
                #    return sum(f(k/n)*G(n*x-k) for k in
↪[ceil(n*a),...,floor(n*b)]) / sum(G(n*x-k) for k in
↪[ceil(n*a),...,floor(n*b)])
                #leftSide = abs(L(n,f,x0)-f(x0))
                leftSide = abs(sum(f(k/n)*G(n*x0-k) for k
↪in [ceil(n*a),...,floor(n*b)]) / sum(G(n*x0-k) for k in
↪[ceil(n*a),...,floor(n*b)]) - f(x0))
                val1 = n
                val2 = leftSide.n()
                val3 = 1/(n^power).n()
                print("          n = "+str(val1), ", left
↪side = "+str(val2), "\n          1/
↪n^("+str(power)+") = "+str(val3), "\n
↪difference = "+str(val3-val2))

```

$x_0 = 1/2$, $\text{Power} = 3/10$, $\text{lamda} = 1/4$, $q = 1/4$

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 7.31673691973467e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.01524844564384e-1
n = 5000 , left side = 2.93115579032222e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.73868453925412e-2

```

$$x$$

```

n = 2000 , left side = 1.38629436111959e-3
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.00870223895238e-1
n = 5000 , left side = 5.54517744448035e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.71254432271253e-2

```

$$x^2$$

```

n = 2000 , left side = 1.39158937464257e-3
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.00864928881715e-1
n = 5000 , left side = 5.55364946611570e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.71245960249618e-2

```

$$x^3$$

```

n = 2000 , left side = 1.04767998407182e-3
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.01208838272285e-1
n = 5000 , left side = 4.17160179930071e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.72628007916433e-2

```

$$x^4$$

```

n = 2000 , left side = 7.01123176443816e-4
           1/n^(3/10) = 1.02256518256357e-1
           difference = 1.01555395079913e-1
n = 5000 , left side = 2.78531814462968e-4
           1/n^(3/10) = 7.76799609715734e-2
           difference = 7.74014291571104e-2

```

$$x^{10}$$

```

n = 2000 , left side = 2.80227731768255e-5
           1/n^(3/10) = 1.02256518256357e-1
           difference = 1.02228495483180e-1
n = 5000 , left side = 1.09803560959030e-5
           1/n^(3/10) = 7.76799609715734e-2
           difference = 7.76689806154775e-2

```

x0 = 1/2, Power = 3/10, lamda = 1/4, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 3.65410972395064e-4
           1/n^(3/10) = 1.02256518256357e-1
           difference = 1.01891107283962e-1
n = 5000 , left side = 1.46489688196305e-4
           1/n^(3/10) = 7.76799609715734e-2
           difference = 7.75334712833771e-2

```

$$x$$

```

n = 2000 , left side = 6.93147180559794e-4
           1/n^(3/10) = 1.02256518256357e-1
           difference = 1.01563371075798e-1

```

n = 5000 , left side = 2.77258872223740e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.74027020993496e-2

$$x^2$$

n = 2000 , left side = 6.97000835040928e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01559517421316e-1
 n = 5000 , left side = 2.77875456940857e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.74020855146325e-2

$$x^3$$

n = 2000 , left side = 5.25648214541347e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01730870041816e-1
 n = 5000 , left side = 2.08869501477027e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.74710914700964e-2

$$x^4$$

n = 2000 , left side = 3.52368823871171e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01904149432486e-1
 n = 5000 , left side = 1.39555255115567e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.75404057164578e-2

$$x^{10}$$

n = 2000 , left side = 1.42225079194256e-5
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02242295748438e-1
 n = 5000 , left side = 5.52404203315975e-6

```
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76744369295402e-2
```

```
-----
x0 = 1/2, Power = 3/10, lamda = 1/4,  q = 1
-----
```

$x^{\frac{1}{3}}$

```
n = 2000 , left side = 1.18994097031422e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02255328315387e-1
n = 5000 , left side = 1.90387245635470e-7
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76797705843277e-2
```

x

```
n = 2000 , left side = 0.000000000000000e0
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256518256357e-1
n = 5000 , left side = 0.000000000000000e0
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76799609715734e-2
```

x^2

```
n = 2000 , left side = 3.37320146703846e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02253145054890e-1
n = 5000 , left side = 5.39712234737255e-7
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76794212593386e-2
```

x^3

```

n = 2000 , left side = 5.05980220055768e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02251458454157e-1
n = 5000 , left side = 8.09568352244661e-7
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76791514032211e-2

```

$$x^4$$

```

n = 2000 , left side = 5.05984931552239e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02251458407042e-1
n = 5000 , left side = 8.09569558321566e-7
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76791514020151e-2

```

$$x^{10}$$

```

n = 2000 , left side = 5.93100186560341e-7
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02255925156171e-1
n = 5000 , left side = 9.48752485302982e-8
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76798660963249e-2

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 3.66279383738788e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.01890238872619e-1
n = 5000 , left side = 1.46628824907791e-4

```


$1/n^{(3/10)} = 7.76799609715734e-2$
 difference = 7.75333321466656e-2

x

n = 2000 , left side = 6.93147191573318e-4
 $1/n^{(3/10)} = 1.02256518256357e-1$
 difference = 1.01563371064784e-1
 n = 5000 , left side = 2.77258876629438e-4
 $1/n^{(3/10)} = 7.76799609715734e-2$
 difference = 7.74027020949439e-2

x^2

n = 2000 , left side = 6.94533444929146e-4
 $1/n^{(3/10)} = 1.02256518256357e-1$
 difference = 1.01561984811428e-1
 n = 5000 , left side = 2.77480677166264e-4
 $1/n^{(3/10)} = 7.76799609715734e-2$
 difference = 7.74024802944071e-2

x^3

n = 2000 , left side = 5.21941990297342e-4
 $1/n^{(3/10)} = 1.02256518256357e-1$
 difference = 1.01734576266060e-1
 n = 5000 , left side = 2.08277000138785e-4
 $1/n^{(3/10)} = 7.76799609715734e-2$
 difference = 7.74716839714346e-2

x^4

n = 2000 , left side = 3.48657415094378e-4
 $1/n^{(3/10)} = 1.02256518256357e-1$
 difference = 1.01907860841263e-1
 n = 5000 , left side = 1.38962422999100e-4

1/n^(3/10) = 7.76799609715734e-2
 difference = 7.75409985485743e-2

x^{10}

n = 2000 , left side = 1.37838066328454e-5
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02242734449724e-1
 n = 5000 , left side = 5.45433431887185e-6
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76745066372545e-2

 x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1/2

$x^{\frac{1}{3}}$

n = 2000 , left side = 1.83022238703923e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02073496017653e-1
 n = 5000 , left side = 7.32956220088621e-5
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76066653495645e-2

x

n = 2000 , left side = 3.46573574531495e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01909944681826e-1
 n = 5000 , left side = 1.38629429813131e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.75413315417603e-2

x^2

```

n = 2000 , left side = 3.47599488122519e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.01908918768235e-1
n = 5000 , left side = 1.38793575987506e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.75411673955859e-2

```

$$x^3$$

```

n = 2000 , left side = 2.61470034692679e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.01995048221665e-1
n = 5000 , left side = 1.04218354559454e-4
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.75757426170139e-2

```

$$x^4$$

```

n = 2000 , left side = 1.74827628399432e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02081690627958e-1
n = 5000 , left side = 6.95610601451069e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76103999114283e-2

```

$$x^{10}$$

```

n = 2000 , left side = 6.95028639569268e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02249567969962e-1
n = 5000 , left side = 2.73651920563241e-6
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76772244523678e-2

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 3.19528048242645e-7
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02256198728309e-1
n = 5000 , left side = 5.11242578271620e-8
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76799098473156e-2

$$x$$

n = 2000 , left side = 5.55111512312578e-17
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02256518256357e-1
n = 5000 , left side = 0.0000000000000000e0
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76799609715734e-2

$$x^2$$

n = 2000 , left side = 9.05800419448433e-7
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02255612455938e-1
n = 5000 , left side = 1.44928067047356e-7
1/n^(3/10) = 7.76799609715734e-2
difference = 7.76798160435063e-2

$$x^3$$

n = 2000 , left side = 1.35870062939469e-6
1/n^(3/10) = 1.02256518256357e-1
difference = 1.02255159555728e-1
n = 5000 , left side = 2.17392100737568e-7

1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76797435794726e-2

$$x^4$$

n = 2000 , left side = 1.35870389414428e-6
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02255159552463e-1
 n = 5000 , left side = 2.17392184295728e-7
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76797435793891e-2

$$x^{10}$$

n = 2000 , left side = 1.59233443226565e-7
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02256359022914e-1
 n = 5000 , left side = 2.54759104940244e-8
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76799354956629e-2

 x0 = 1/2, Power = 3/10, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 1.83159113621878e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02073359142735e-1
 n = 5000 , left side = 7.32982158705431e-5
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76066627557028e-2

$$x$$

n = 2000 , left side = 3.46421360206572e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01910096896151e-1
 n = 5000 , left side = 1.38568544082562e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.75413924274908e-2

$$x^2$$

n = 2000 , left side = 3.46830228773298e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01909688027584e-1
 n = 5000 , left side = 1.38633963053358e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.75413270085200e-2

$$x^3$$

n = 2000 , left side = 2.60429665156547e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.01996088591201e-1
 n = 5000 , left side = 1.04024558415738e-4
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.75759364131576e-2

$$x^4$$

n = 2000 , left side = 1.73824667772216e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02082693588585e-1
 n = 5000 , left side = 6.93824443059460e-5
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76105785272674e-2

$$x^{10}$$

```

n = 2000 , left side = 6.83823608409119e-6
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02249680020273e-1
n = 5000 , left side = 2.71793687885765e-6
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76772430346945e-2

```

x0 = 1/2, Power = 3/10, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 9.16500819068133e-5
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02164868174451e-1
n = 5000 , left side = 3.66870091477178e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76432739624257e-2

```

$$x$$

```

n = 2000 , left side = 1.73420272625346e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02083097983732e-1
n = 5000 , left side = 6.93681090501164e-5
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76105928625233e-2

```

$$x^2$$

```

n = 2000 , left side = 1.73739151737184e-4
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02082779104620e-1
n = 5000 , left side = 6.94191297080926e-5

```

1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76105418418653e-2

$$x^3$$

n = 2000 , left side = 1.30543678276612e-4
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02125974578081e-1
 n = 5000 , left side = 5.21026227034427e-5
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76278583488699e-2

$$x^4$$

n = 2000 , left side = 8.71887656059556e-5
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02169329490751e-1
 n = 5000 , left side = 3.47606053786725e-5
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76452003661947e-2

$$x^{10}$$

n = 2000 , left side = 3.44331424903836e-6
 1/n^(3/10) = 1.02256518256357e-1
 difference = 1.02253074942108e-1
 n = 5000 , left side = 1.36382369174317e-6
 1/n^(3/10) = 7.76799609715734e-2
 difference = 7.76785971478816e-2

 x0 = 1/2, Power = 3/10, lamda = 1, q = 1

$$x^{\frac{1}{3}}$$


```

n = 2000 , left side = 1.02019013792365e-7
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256416237344e-1
n = 5000 , left side = 1.63230216010390e-8
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76799446485518e-2

```

x

```

n = 2000 , left side = 0.000000000000000e0
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256518256357e-1
n = 5000 , left side = 0.000000000000000e0
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76799609715734e-2

```

x^2

```

n = 2000 , left side = 2.89205348558141e-7
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256229051009e-1
n = 5000 , left side = 4.62728558248138e-8
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76799146987176e-2

```

x^3

```

n = 2000 , left side = 4.33808022975990e-7
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256084448334e-1
n = 5000 , left side = 6.94092836817095e-8
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76798915622897e-2

```

x^4

```

n = 2000 , left side = 4.33808315547513e-7
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256084448042e-1
n = 5000 , left side = 6.94092910924482e-8
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76798915622823e-2

```

$$x^{10}$$

```

n = 2000 , left side = 5.08378382214642e-8
          1/n^(3/10) = 1.02256518256357e-1
          difference = 1.02256467418519e-1
n = 5000 , left side = 8.13392532625010e-9
          1/n^(3/10) = 7.76799609715734e-2
          difference = 7.76799528376481e-2

```

x0 = 1/2, Power = 1/2, lamda = 1/4, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 7.31673691973467e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.16290060830244e-2
n = 5000 , left side = 2.93115579032222e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.38490200446987e-2

```

$$x$$

```

n = 2000 , left side = 1.38629436111959e-3
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.09743854138783e-2
n = 5000 , left side = 5.54517744448035e-4

```

$1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.35876178792829e-2

x^2

n = 2000 , left side = 1.39158937464257e-3
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.09690904003553e-2
n = 5000 , left side = 5.55364946611570e-4
 $1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.35867706771194e-2

x^3

n = 2000 , left side = 1.04767998407182e-3
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.13129997909261e-2
n = 5000 , left side = 4.17160179930071e-4
 $1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.37249754438009e-2

x^4

n = 2000 , left side = 7.01123176443816e-4
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.16595565985541e-2
n = 5000 , left side = 2.78531814462968e-4
 $1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.38636038092680e-2

x^{10}

n = 2000 , left side = 2.80227731768255e-5
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.23326570018211e-2
n = 5000 , left side = 1.09803560959030e-5
 $1/n^{(1/2)} = 1.41421356237310e-2$

difference = 1.41311552676350e-2

x0 = 1/2, Power = 1/2, lamda = 1/4, q = 1/2

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 3.65410972395064e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.19952688026028e-2
n = 5000 , left side = 1.46489688196305e-4
1/n^(1/2) = 1.41421356237310e-2
difference = 1.39956459355346e-2

$$x$$

n = 2000 , left side = 6.93147180559794e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.16675325944381e-2
n = 5000 , left side = 2.77258872223740e-4
1/n^(1/2) = 1.41421356237310e-2
difference = 1.38648767515072e-2

$$x^2$$

n = 2000 , left side = 6.97000835040928e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.16636789399570e-2
n = 5000 , left side = 2.77875456940857e-4
1/n^(1/2) = 1.41421356237310e-2
difference = 1.38642601667901e-2

$$x^3$$

```

n = 2000 , left side = 5.25648214541347e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.18350315604566e-2
n = 5000 , left side = 2.08869501477027e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.39332661222539e-2

```

$$x^4$$

```

n = 2000 , left side = 3.52368823871171e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.20083109511267e-2
n = 5000 , left side = 1.39555255115567e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.40025803686154e-2

```

$$x^{10}$$

```

n = 2000 , left side = 1.42225079194256e-5
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23464572670785e-2
n = 5000 , left side = 5.52404203315975e-6
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41366115816978e-2

```

x0 = 1/2, Power = 1/2, lamda = 1/4, q = 1

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 1.18994097031422e-6
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23594898340276e-2
n = 5000 , left side = 1.90387245635470e-7

```

$1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.41419452364853e-2

x

n = 2000 , left side = 0.0000000000000000e0
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.23606797749979e-2
n = 5000 , left side = 0.0000000000000000e0
 $1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.41421356237310e-2

x^2

n = 2000 , left side = 3.37320146703846e-6
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.23573065735309e-2
n = 5000 , left side = 5.39712234737255e-7
 $1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.41415959114962e-2

x^3

n = 2000 , left side = 5.05980220055768e-6
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.23556199727973e-2
n = 5000 , left side = 8.09568352244661e-7
 $1/n^{(1/2)} = 1.41421356237310e-2$
difference = 1.41413260553787e-2

x^4

n = 2000 , left side = 5.05984931552239e-6
 $1/n^{(1/2)} = 2.23606797749979e-2$
difference = 2.23556199256824e-2
n = 5000 , left side = 8.09569558321566e-7

1/n^(1/2) = 1.41421356237310e-2
 difference = 1.41413260541726e-2

$$x^{10}$$

n = 2000 , left side = 5.93100186560341e-7
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.23600866748113e-2
 n = 5000 , left side = 9.48752485302982e-8
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.41420407484824e-2

 x0 = 1/2, Power = 1/2, lamda = 1/2, q = 1/4

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 3.66279383738788e-4
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.19944003912591e-2
 n = 5000 , left side = 1.46628824907791e-4
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.39955067988232e-2

$$x$$

n = 2000 , left side = 6.93147191573318e-4
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.16675325834246e-2
 n = 5000 , left side = 2.77258876629438e-4
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.38648767471015e-2

$$x^2$$

n = 2000 , left side = 6.94533444929146e-4
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.16661463300688e-2
 n = 5000 , left side = 2.77480677166264e-4
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.38646549465647e-2

$$x^3$$

n = 2000 , left side = 5.21941990297342e-4
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.18387377847006e-2
 n = 5000 , left side = 2.08277000138785e-4
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.39338586235922e-2

$$x^4$$

n = 2000 , left side = 3.48657415094378e-4
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.20120223599035e-2
 n = 5000 , left side = 1.38962422999100e-4
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.40031732007319e-2

$$x^{10}$$

n = 2000 , left side = 1.37838066328454e-5
 1/n^(1/2) = 2.23606797749979e-2
 difference = 2.23468959683651e-2
 n = 5000 , left side = 5.45433431887185e-6
 1/n^(1/2) = 1.41421356237310e-2
 difference = 1.41366812894121e-2

x0 = 1/2, Power = 1/2, lamda = 1/2, q = 1/2

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 1.83022238703923e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.21776575362940e-2
n = 5000 , left side = 7.32956220088621e-5
1/n^(1/2) = 1.41421356237310e-2
difference = 1.40688400017221e-2

$$x$$

n = 2000 , left side = 3.46573574531495e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.20141062004664e-2
n = 5000 , left side = 1.38629429813131e-4
1/n^(1/2) = 1.41421356237310e-2
difference = 1.40035061939178e-2

$$x^2$$

n = 2000 , left side = 3.47599488122519e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.20130802868754e-2
n = 5000 , left side = 1.38793575987506e-4
1/n^(1/2) = 1.41421356237310e-2
difference = 1.40033420477434e-2

$$x^3$$

n = 2000 , left side = 2.61470034692679e-4
1/n^(1/2) = 2.23606797749979e-2
difference = 2.20992097403052e-2
n = 5000 , left side = 1.04218354559454e-4

$1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.40379172691715e-2

$$x^4$$

n = 2000 , left side = 1.74827628399432e-4
 $1/n^{(1/2)} = 2.23606797749979e-2$
 difference = 2.21858521465985e-2
 n = 5000 , left side = 6.95610601451069e-5
 $1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.40725745635858e-2

$$x^{10}$$

n = 2000 , left side = 6.95028639569268e-6
 $1/n^{(1/2)} = 2.23606797749979e-2$
 difference = 2.23537294886022e-2
 n = 5000 , left side = 2.73651920563241e-6
 $1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.41393991045253e-2

 $x_0 = 1/2$, Power = 1/2, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 3.19528048242645e-7
 $1/n^{(1/2)} = 2.23606797749979e-2$
 difference = 2.23603602469497e-2
 n = 5000 , left side = 5.11242578271620e-8
 $1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.41420844994731e-2

$$x$$

```

n = 2000 , left side = 5.55111512312578e-17
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23606797749978e-2
n = 5000 , left side = 0.000000000000000e0
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41421356237310e-2

```

$$x^2$$

```

n = 2000 , left side = 9.05800419448433e-7
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23597739745784e-2
n = 5000 , left side = 1.44928067047356e-7
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41419906956639e-2

```

$$x^3$$

```

n = 2000 , left side = 1.35870062939469e-6
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23593210743685e-2
n = 5000 , left side = 2.17392100737568e-7
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41419182316302e-2

```

$$x^4$$

```

n = 2000 , left side = 1.35870389414428e-6
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23593210711038e-2
n = 5000 , left side = 2.17392184295728e-7
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41419182315467e-2

```

$$x^{10}$$

```

n = 2000 , left side = 1.59233443226565e-7
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23605205415547e-2
n = 5000 , left side = 2.54759104940244e-8
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41421101478205e-2

```

x0 = 1/2, Power = 1/2, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 1.83159113621878e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.21775206613760e-2
n = 5000 , left side = 7.32982158705431e-5
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.40688374078604e-2

```

$$x$$

```

n = 2000 , left side = 3.46421360206572e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.20142584147913e-2
n = 5000 , left side = 1.38568544082562e-4
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.40035670796484e-2

```

$$x^2$$

```

n = 2000 , left side = 3.46830228773298e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.20138495462246e-2
n = 5000 , left side = 1.38633963053358e-4

```

$1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.40035016606776e-2

$$x^3$$

n = 2000 , left side = 2.60429665156547e-4
 $1/n^{(1/2)} = 2.23606797749979e-2$
 difference = 2.21002501098413e-2
 n = 5000 , left side = 1.04024558415738e-4
 $1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.40381110653152e-2

$$x^4$$

n = 2000 , left side = 1.73824667772216e-4
 $1/n^{(1/2)} = 2.23606797749979e-2$
 difference = 2.21868551072257e-2
 n = 5000 , left side = 6.93824443059460e-5
 $1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.40727531794250e-2

$$x^{10}$$

n = 2000 , left side = 6.83823608409119e-6
 $1/n^{(1/2)} = 2.23606797749979e-2$
 difference = 2.23538415389138e-2
 n = 5000 , left side = 2.71793687885765e-6
 $1/n^{(1/2)} = 1.41421356237310e-2$
 difference = 1.41394176868521e-2

 x0 = 1/2, Power = 1/2, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 9.16500819068133e-5
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.22690296930911e-2
n = 5000 , left side = 3.66870091477178e-5
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41054486145832e-2

```

x

```

n = 2000 , left side = 1.73420272625346e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.21872595023726e-2
n = 5000 , left side = 6.93681090501164e-5
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.40727675146808e-2

```

x^2

```

n = 2000 , left side = 1.73739151737184e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.21869406232607e-2
n = 5000 , left side = 6.94191297080926e-5
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.40727164940229e-2

```

x^3

```

n = 2000 , left side = 1.30543678276612e-4
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.22301360967213e-2
n = 5000 , left side = 5.21026227034427e-5
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.40900330010275e-2

```

x^4

```

n = 2000 , left side = 8.71887656059556e-5
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.22734910093919e-2
n = 5000 , left side = 3.47606053786725e-5
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41073750183523e-2

```

$$x^{10}$$

```

n = 2000 , left side = 3.44331424903836e-6
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23572364607489e-2
n = 5000 , left side = 1.36382369174317e-6
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41407718000392e-2

```

x0 = 1/2, Power = 1/2, lamda = 1, q = 1

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 1.02019013792365e-7
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23605777559841e-2
n = 5000 , left side = 1.63230216010390e-8
          1/n^(1/2) = 1.41421356237310e-2
          difference = 1.41421193007093e-2

```

$$x$$

```

n = 2000 , left side = 0.000000000000000e0
          1/n^(1/2) = 2.23606797749979e-2
          difference = 2.23606797749979e-2
n = 5000 , left side = 0.000000000000000e0

```

$1/n^{(1/2)} = 1.41421356237310e-2$
 $\text{difference} = 1.41421356237310e-2$

$$x^2$$

$n = 2000$, left side = $2.89205348558141e-7$
 $1/n^{(1/2)} = 2.23606797749979e-2$
 $\text{difference} = 2.23603905696493e-2$
 $n = 5000$, left side = $4.62728558248138e-8$
 $1/n^{(1/2)} = 1.41421356237310e-2$
 $\text{difference} = 1.41420893508751e-2$

$$x^3$$

$n = 2000$, left side = $4.33808022975990e-7$
 $1/n^{(1/2)} = 2.23606797749979e-2$
 $\text{difference} = 2.23602459669749e-2$
 $n = 5000$, left side = $6.94092836817095e-8$
 $1/n^{(1/2)} = 1.41421356237310e-2$
 $\text{difference} = 1.41420662144473e-2$

$$x^4$$

$n = 2000$, left side = $4.33808315547513e-7$
 $1/n^{(1/2)} = 2.23606797749979e-2$
 $\text{difference} = 2.23602459666823e-2$
 $n = 5000$, left side = $6.94092910924482e-8$
 $1/n^{(1/2)} = 1.41421356237310e-2$
 $\text{difference} = 1.41420662144399e-2$

$$x^{10}$$

$n = 2000$, left side = $5.08378382214642e-8$
 $1/n^{(1/2)} = 2.23606797749979e-2$
 $\text{difference} = 2.23606289371597e-2$
 $n = 5000$, left side = $8.13392532625010e-9$
 $1/n^{(1/2)} = 1.41421356237310e-2$

difference = 1.41421274898056e-2

x0 = 1/2, Power = 7/10, lamda = 1/4, q = 1/4

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 7.31673691973467e-4
1/n^(7/10) = 4.88966384271464e-3
difference = 4.15799015074118e-3
n = 5000 , left side = 2.93115579032222e-4
1/n^(7/10) = 2.57466658709045e-3
difference = 2.28155100805823e-3

$$x$$

n = 2000 , left side = 1.38629436111959e-3
1/n^(7/10) = 4.88966384271464e-3
difference = 3.50336948159506e-3
n = 5000 , left side = 5.54517744448035e-4
1/n^(7/10) = 2.57466658709045e-3
difference = 2.02014884264241e-3

$$x^2$$

n = 2000 , left side = 1.39158937464257e-3
1/n^(7/10) = 4.88966384271464e-3
difference = 3.49807446807208e-3
n = 5000 , left side = 5.55364946611570e-4
1/n^(7/10) = 2.57466658709045e-3
difference = 2.01930164047888e-3

$$x^3$$

```

n = 2000 , left side = 1.04767998407182e-3
          1/n^(7/10) = 4.88966384271464e-3
          difference = 3.84198385864282e-3
n = 5000 , left side = 4.17160179930071e-4
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.15750640716038e-3

```

$$x^4$$

```

n = 2000 , left side = 7.01123176443816e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.18854066627083e-3
n = 5000 , left side = 2.78531814462968e-4
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.29613477262748e-3

```

$$x^{10}$$

```

n = 2000 , left side = 2.80227731768255e-5
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.86164106953782e-3
n = 5000 , left side = 1.09803560959030e-5
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.56368623099455e-3

```

x0 = 1/2, Power = 7/10, lamda = 1/4, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 3.65410972395064e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.52425287031958e-3
n = 5000 , left side = 1.46489688196305e-4

```

$1/n^{(7/10)} = 2.57466658709045e-3$
difference = 2.42817689889414e-3

x

n = 2000 , left side = 6.93147180559794e-4
 $1/n^{(7/10)} = 4.88966384271464e-3$
difference = 4.19651666215485e-3
n = 5000 , left side = 2.77258872223740e-4
 $1/n^{(7/10)} = 2.57466658709045e-3$
difference = 2.29740771486671e-3

x^2

n = 2000 , left side = 6.97000835040928e-4
 $1/n^{(7/10)} = 4.88966384271464e-3$
difference = 4.19266300767372e-3
n = 5000 , left side = 2.77875456940857e-4
 $1/n^{(7/10)} = 2.57466658709045e-3$
difference = 2.29679113014959e-3

x^3

n = 2000 , left side = 5.25648214541347e-4
 $1/n^{(7/10)} = 4.88966384271464e-3$
difference = 4.36401562817330e-3
n = 5000 , left side = 2.08869501477027e-4
 $1/n^{(7/10)} = 2.57466658709045e-3$
difference = 2.36579708561342e-3

x^4

n = 2000 , left side = 3.52368823871171e-4
 $1/n^{(7/10)} = 4.88966384271464e-3$
difference = 4.53729501884347e-3
n = 5000 , left side = 1.39555255115567e-4

```
1/n^(7/10) = 2.57466658709045e-3
difference = 2.43511133197488e-3
```

x^{10}

```
n = 2000 , left side = 1.42225079194256e-5
1/n^(7/10) = 4.88966384271464e-3
difference = 4.87544133479522e-3
n = 5000 , left side = 5.52404203315975e-6
1/n^(7/10) = 2.57466658709045e-3
difference = 2.56914254505729e-3
```

```
x0 = 1/2, Power = 7/10, lamda = 1/4, q = 1
```

$x^{\frac{1}{3}}$

```
n = 2000 , left side = 1.18994097031422e-6
1/n^(7/10) = 4.88966384271464e-3
difference = 4.88847390174433e-3
n = 5000 , left side = 1.90387245635470e-7
1/n^(7/10) = 2.57466658709045e-3
difference = 2.57447619984481e-3
```

x

```
n = 2000 , left side = 0.000000000000000e0
1/n^(7/10) = 4.88966384271464e-3
difference = 4.88966384271464e-3
n = 5000 , left side = 0.000000000000000e0
1/n^(7/10) = 2.57466658709045e-3
difference = 2.57466658709045e-3
```

x^2

```

n = 2000 , left side = 3.37320146703846e-6
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88629064124761e-3
n = 5000 , left side = 5.39712234737255e-7
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57412687485571e-3

```

$$x^3$$

```

n = 2000 , left side = 5.05980220055768e-6
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88460404051409e-3
n = 5000 , left side = 8.09568352244661e-7
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57385701873820e-3

```

$$x^4$$

```

n = 2000 , left side = 5.05984931552239e-6
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88460399339912e-3
n = 5000 , left side = 8.09569558321566e-7
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57385701753213e-3

```

$$x^{10}$$

```

n = 2000 , left side = 5.93100186560341e-7
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88907074252808e-3
n = 5000 , left side = 9.48752485302982e-8
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57457171184192e-3

```

x0 = 1/2, Power = 7/10, lamda = 1/2, q = 1/4

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 3.66279383738788e-4
1/n^(7/10) = 4.88966384271464e-3
difference = 4.52338445897586e-3
n = 5000 , left side = 1.46628824907791e-4
1/n^(7/10) = 2.57466658709045e-3
difference = 2.42803776218266e-3

$$x$$

n = 2000 , left side = 6.93147191573318e-4
1/n^(7/10) = 4.88966384271464e-3
difference = 4.19651665114133e-3
n = 5000 , left side = 2.77258876629438e-4
1/n^(7/10) = 2.57466658709045e-3
difference = 2.29740771046101e-3

$$x^2$$

n = 2000 , left side = 6.94533444929146e-4
1/n^(7/10) = 4.88966384271464e-3
difference = 4.19513039778550e-3
n = 5000 , left side = 2.77480677166264e-4
1/n^(7/10) = 2.57466658709045e-3
difference = 2.29718590992418e-3

$$x^3$$

n = 2000 , left side = 5.21941990297342e-4
1/n^(7/10) = 4.88966384271464e-3
difference = 4.36772185241730e-3
n = 5000 , left side = 2.08277000138785e-4

1/n^(7/10) = 2.57466658709045e-3
 difference = 2.36638958695166e-3

$$x^4$$

n = 2000 , left side = 3.48657415094378e-4
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.54100642762027e-3
 n = 5000 , left side = 1.38962422999100e-4
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.43570416409135e-3

$$x^{10}$$

n = 2000 , left side = 1.37838066328454e-5
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.87588003608180e-3
 n = 5000 , left side = 5.45433431887185e-6
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.56921225277158e-3

 x0 = 1/2, Power = 7/10, lamda = 1/2, q = 1/2

$$x^{\frac{1}{3}}$$

n = 2000 , left side = 1.83022238703923e-4
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.70664160401072e-3
 n = 5000 , left side = 7.32956220088621e-5
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.50137096508159e-3

$$x$$

n = 2000 , left side = 3.46573574531495e-4
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.54309026818315e-3
 n = 5000 , left side = 1.38629429813131e-4
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.43603715727732e-3

$$x^2$$

n = 2000 , left side = 3.47599488122519e-4
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.54206435459213e-3
 n = 5000 , left side = 1.38793575987506e-4
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.43587301110294e-3

$$x^3$$

n = 2000 , left side = 2.61470034692679e-4
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.62819380802197e-3
 n = 5000 , left side = 1.04218354559454e-4
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.47044823253099e-3

$$x^4$$

n = 2000 , left side = 1.74827628399432e-4
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.71483621431521e-3
 n = 5000 , left side = 6.95610601451069e-5
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.50510552694534e-3

$$x^{10}$$


```

n = 2000 , left side = 6.95028639569268e-6
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88271355631895e-3
n = 5000 , left side = 2.73651920563241e-6
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57193006788482e-3

```

x0 = 1/2, Power = 7/10, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 3.19528048242645e-7
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88934431466640e-3
n = 5000 , left side = 5.11242578271620e-8
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57461546283262e-3

```

$$x$$

```

n = 2000 , left side = 5.55111512312578e-17
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88966384271459e-3
n = 5000 , left side = 0.000000000000000e0
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57466658709045e-3

```

$$x^2$$

```

n = 2000 , left side = 9.05800419448433e-7
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88875804229520e-3
n = 5000 , left side = 1.44928067047356e-7

```

1/n^(7/10) = 2.57466658709045e-3
 difference = 2.57452165902340e-3

$$x^3$$

n = 2000 , left side = 1.35870062939469e-6
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.88830514208525e-3
 n = 5000 , left side = 2.17392100737568e-7
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.57444919498971e-3

$$x^4$$

n = 2000 , left side = 1.35870389414428e-6
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.88830513882050e-3
 n = 5000 , left side = 2.17392184295728e-7
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.57444919490615e-3

$$x^{10}$$

n = 2000 , left side = 1.59233443226565e-7
 1/n^(7/10) = 4.88966384271464e-3
 difference = 4.88950460927142e-3
 n = 5000 , left side = 2.54759104940244e-8
 1/n^(7/10) = 2.57466658709045e-3
 difference = 2.57464111117995e-3

 x0 = 1/2, Power = 7/10, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 1.83159113621878e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.70650472909277e-3
n = 5000 , left side = 7.32982158705431e-5
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.50136837121991e-3

```

x

```

n = 2000 , left side = 3.46421360206572e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.54324248250807e-3
n = 5000 , left side = 1.38568544082562e-4
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.43609804300789e-3

```

x^2

```

n = 2000 , left side = 3.46830228773298e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.54283361394135e-3
n = 5000 , left side = 1.38633963053358e-4
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.43603262403709e-3

```

x^3

```

n = 2000 , left side = 2.60429665156547e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.62923417755810e-3
n = 5000 , left side = 1.04024558415738e-4
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.47064202867471e-3

```

x^4

```

n = 2000 , left side = 1.73824667772216e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.71583917494243e-3
n = 5000 , left side = 6.93824443059460e-5
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.50528414278450e-3

```

$$x^{10}$$

```

n = 2000 , left side = 6.83823608409119e-6
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88282560663055e-3
n = 5000 , left side = 2.71793687885765e-6
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57194865021159e-3

```

x0 = 1/2, Power = 7/10, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 2000 , left side = 9.16500819068133e-5
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.79801376080783e-3
n = 5000 , left side = 3.66870091477178e-5
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.53797957794273e-3

```

$$x$$

```

n = 2000 , left side = 1.73420272625346e-4
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.71624357008930e-3
n = 5000 , left side = 6.93681090501164e-5

```

$1/n^{(7/10)} = 2.57466658709045e-3$
 difference = 2.50529847804033e-3

x^2

n = 2000 , left side = 1.73739151737184e-4
 $1/n^{(7/10)} = 4.88966384271464e-3$
 difference = 4.71592469097746e-3
 n = 5000 , left side = 6.94191297080926e-5
 $1/n^{(7/10)} = 2.57466658709045e-3$
 difference = 2.50524745738236e-3

x^3

n = 2000 , left side = 1.30543678276612e-4
 $1/n^{(7/10)} = 4.88966384271464e-3$
 difference = 4.75912016443803e-3
 n = 5000 , left side = 5.21026227034427e-5
 $1/n^{(7/10)} = 2.57466658709045e-3$
 difference = 2.52256396438701e-3

x^4

n = 2000 , left side = 8.71887656059556e-5
 $1/n^{(7/10)} = 4.88966384271464e-3$
 difference = 4.80247507710869e-3
 n = 5000 , left side = 3.47606053786725e-5
 $1/n^{(7/10)} = 2.57466658709045e-3$
 difference = 2.53990598171178e-3

x^{10}

n = 2000 , left side = 3.44331424903836e-6
 $1/n^{(7/10)} = 4.88966384271464e-3$
 difference = 4.88622052846561e-3
 n = 5000 , left side = 1.36382369174317e-6
 $1/n^{(7/10)} = 2.57466658709045e-3$

difference = 2.57330276339871e-3

x0 = 1/2, Power = 7/10, lamda = 1, q = 1

$x^{\frac{1}{3}}$

n = 2000 , left side = 1.02019013792365e-7
1/n^(7/10) = 4.88966384271464e-3
difference = 4.88956182370085e-3
n = 5000 , left side = 1.63230216010390e-8
1/n^(7/10) = 2.57466658709045e-3
difference = 2.57465026406885e-3

x

n = 2000 , left side = 0.000000000000000e0
1/n^(7/10) = 4.88966384271464e-3
difference = 4.88966384271464e-3
n = 5000 , left side = 0.000000000000000e0
1/n^(7/10) = 2.57466658709045e-3
difference = 2.57466658709045e-3

x^2

n = 2000 , left side = 2.89205348558141e-7
1/n^(7/10) = 4.88966384271464e-3
difference = 4.88937463736609e-3
n = 5000 , left side = 4.62728558248138e-8
1/n^(7/10) = 2.57466658709045e-3
difference = 2.57462031423462e-3

x^3

```

n = 2000 , left side = 4.33808022975990e-7
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88923003469167e-3
n = 5000 , left side = 6.94092836817095e-8
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57459717780677e-3

```

$$x^4$$

```

n = 2000 , left side = 4.33808315547513e-7
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88923003439910e-3
n = 5000 , left side = 6.94092910924482e-8
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57459717779936e-3

```

$$x^{10}$$

```

n = 2000 , left side = 5.08378382214642e-8
          1/n^(7/10) = 4.88966384271464e-3
          difference = 4.88961300487642e-3
n = 5000 , left side = 8.13392532625010e-9
          1/n^(7/10) = 2.57466658709045e-3
          difference = 2.57465845316512e-3

```

5 Real-valued neural network approximation based on the q-deformed and λ -parametrized Hyperbolic Tangent - part 3

```
[ ]: RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1]    #deformation parameter lamda over (0,
    ↪ 1] - these are the beta values in the formula
qs = [1/4, 1/2, 1]    #deformation coefficient

funcs = [sin(x), cos(x)]    #choice of functions
a = -pi    #the interval
b = pi    #the interval
x0s= [pi/4, pi/2, 3*pi/4]

#####
for x0 in x0s:
    #####
        for power in powers:    #going over various powers for 1/
            ↪ n^power

            ↪
            ↪#####
                for lamda in lamdas:    #going over each lamda value

                ↪
                ↪#####
                    for q in qs:    #going over each q value

                    ↪
                    ↪#####
                        print()
                        print()

                    ↪
                    ↪#####
                        print("-----")
                        print("x0 = " + str(x0)+", Power = "+
            ↪str(power)+ ", lamda = "+ str(lamda) + ", q = " + str(q))
```



```

    print("-----")

    #the activation function
    phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
    (e^(lamda*x)+q*e^(-lamda*x)) #formula 18.1

    #
    G(x) = 1/4*(phi(x+1) - phi(x-1)) #formula
    18.9

    #####
    for i in range(len(funcs)):
        #####
        f(x)=funcs[i]
        show(f(x))
        for n in [10000, 20000]:
            #def L(n, f, x): #real-valued
            # return sum(f(k/n)*G(n*x-k) for k
            in [ceil(n*a),...,floor(n*b)])/sum(G(n*x-k) for k in
            [ceil(n*a),...,floor(n*b)])
            #leftSide = abs(L(n,f,x0)-f(x0))
            leftSide = abs(sum(f(k/n)*G(n*x0-k)
            for k in [ceil(n*a),...,floor(n*b)])/sum(G(n*x0-k) for k in
            [ceil(n*a),...,floor(n*b)])-f(x0))
            val1 = n
            val2 = leftSide.n()
            val3 = 1/(n^power).n()
            print("n = "+str(val1), ",
            left side = "+str(val2), "\n
            n^("+str(power)+") = "+str(val3), "\n
            difference = "+str(val3-val2))

```

x0 = 1/4*pi, Power = 3/10, lamda = 1/4, q = 1/4

$\sin(x)$

n = 10000 , left side = 1.95976730160918e-4
1/n^(3/10) = 6.30957344480193e-2
difference = 6.28997577178584e-2
n = 20000 , left side = 9.80070916810627e-5
1/n^(3/10) = 5.12496615052604e-2
difference = 5.11516544135793e-2

$\cos(x)$

n = 10000 , left side = 1.96126495751847e-4
1/n^(3/10) = 6.30957344480193e-2
difference = 6.28996079522675e-2
n = 20000 , left side = 9.80445330771573e-5
1/n^(3/10) = 5.12496615052604e-2
difference = 5.11516169721832e-2

x0 = 1/4*pi, Power = 3/10, lamda = 1/4, q = 1/2

$\sin(x)$

n = 10000 , left side = 9.79713085184075e-5
1/n^(3/10) = 6.30957344480193e-2
difference = 6.29977631395009e-2

```

[ ]: RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1]    #deformation parameter lamda over (0,
    ↪ 1] - these are the beta values in the formula
qs = [1/4, 1/2, 1]    #deformation coefficient

funcs = [x^(1/3), x, x^2, x^3, x^4, x^10]    #choice of
    ↪ functions
a = -1    #the interval
b = 1    #the interval
x0=1/2

for power in powers:
    ↪
    ↪#####
    for lamda in lamdas:    #going over each lamda value
        ↪
        ↪#####
        for q in qs:        #going over each q value
            ↪
            ↪#####
            print()
            print()
            ↪
            ↪print("-----")
            print("x0 = " + str(x0)+", Power = "+ str(power)+
            ↪", lamda = "+ str(lamda) + ", q = " + str(q))
            ↪
            ↪print("-----")

            #the activation function
            phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
            ↪(e^(lamda*x)+q*e^(-lamda*x))    #formula 18.1

```

```

#
G(x) = 1/4*(phi(x+1) - phi(x-1))      #formula 18.9

    □
↳ #####
    for i in range(len(funcs)):
        □
↳ #####
        f(x)=funcs[i]
        show(f(x))
        for n in [10000, 20000]:
            #def L(n, f, x):      #real-valued linear
↳ neural network operators
            #      return sum(f(k/n)*G(n*x-k) for k in
↳ [ceil(n*a),...,floor(n*b)]) / sum(G(n*x-k) for k in
↳ [ceil(n*a),...,floor(n*b)])
            #leftSide = abs(L(n,f,x0)-f(x0))
            leftSide = abs(sum(f(k/n)*G(n*x0-k) for k
↳ in [ceil(n*a),...,floor(n*b)]) / sum(G(n*x0-k) for k in
↳ [ceil(n*a),...,floor(n*b)]) - f(x0))
            val1 = n
            val2 = leftSide.n()
            val3 = 1/(n^power).n()
            print("          n = "+str(val1), ", left
↳ side = "+str(val2), "\n          1/
↳ n^("+str(power)+") = "+str(val3), "\n
↳ difference = "+str(val3-val2))

```

$x_0 = 1/2$, $\text{Power} = 3/10$, $\text{lamda} = 1/4$, $q = 1/4$

$$x^{\frac{1}{3}}$$

```

n = 10000 , left side = 1.46632346878395e-4
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.29491021011409e-2
n = 20000 , left side = 7.33348323127547e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.11763266729476e-2

```

$$x$$

```

n = 10000 , left side = 2.77258872223962e-4
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.28184755757954e-2
n = 20000 , left side = 1.38629436111981e-4
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.11110320691484e-2

```

$$x^2$$

```

n = 10000 , left side = 2.77470672764790e-4
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.28182637752545e-2
n = 20000 , left side = 1.38682386247202e-4
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.11109791190132e-2

```

$$x^3$$

```

n = 10000 , left side = 2.08261988522945e-4
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.28874724594964e-2
n = 20000 , left side = 1.04051518979742e-4
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.11456099862806e-2

```

$$x^4$$

```
n = 10000 , left side = 1.38947404154099e-4
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.29567870438652e-2
n = 20000 , left side = 6.93941766531508e-5
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.11802673286072e-2
```

$$x^{10}$$

```
n = 10000 , left side = 5.45256858194539e-6
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30902818794374e-2
n = 20000 , left side = 2.71692949521066e-6
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12469445757652e-2
```

x0 = 1/2, Power = 3/10, lamda = 1/4, q = 1/2

$$x^{\frac{1}{3}}$$

```
n = 10000 , left side = 7.32991512724102e-5
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30224352967469e-2
n = 20000 , left side = 3.66631610100310e-5
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12129983442503e-2
```

$$x$$

```
n = 10000 , left side = 1.38629436112869e-4
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.29571050119065e-2
```

n = 20000 , left side = 6.93147180559350e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.11803467872044e-2

$$x^2$$

n = 10000 , left side = 1.38783582291413e-4
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.29569508657279e-2
 n = 20000 , left side = 6.93532546008901e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.11803082506595e-2

$$x^3$$

n = 10000 , left side = 1.04203355131977e-4
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.29915310928874e-2
 n = 20000 , left side = 5.20438507066179e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.11976176545538e-2

$$x^4$$

n = 10000 , left side = 6.95460549745980e-5
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30261883930447e-2
 n = 20000 , left side = 3.47151785462102e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12149463267142e-2

$$x^{10}$$

n = 10000 , left side = 2.73475758799874e-6
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30929996904313e-2
 n = 20000 , left side = 1.36058399549210e-6

```
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12483009212649e-2
```

```
-----
x0 = 1/2, Power = 3/10, lamda = 1/4,  q = 1
-----
```

$$x^{\frac{1}{3}}$$

```
n = 10000 , left side = 4.75966933644045e-8
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30956868513260e-2
n = 20000 , left side = 1.18991648756506e-8
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12496496060955e-2
```

$$x$$

```
n = 10000 , left side = 0.0000000000000000e0
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30957344480193e-2
n = 20000 , left side = 0.0000000000000000e0
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12496615052604e-2
```

$$x^2$$

```
n = 10000 , left side = 1.34928058670436e-7
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30955995199607e-2
n = 20000 , left side = 3.37320146259756e-8
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12496277732458e-2
```

$$x^3$$


```

n = 10000 , left side = 2.02392088061165e-7
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30955320559313e-2
n = 20000 , left side = 5.05980219667190e-8
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12496109072384e-2

```

$$x^4$$

```

n = 10000 , left side = 2.02392163403675e-7
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30955320558559e-2
n = 20000 , left side = 5.05980266712891e-8
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12496109072337e-2

```

$$x^{10}$$

```

n = 10000 , left side = 2.37180713688959e-8
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30957107299480e-2
n = 20000 , left side = 5.92947202253911e-9
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12496555757884e-2

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10000 , left side = 7.33339519934928e-5
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30224004960258e-2
n = 20000 , left side = 3.66718634812191e-5

```

$1/n^{(3/10)} = 5.12496615052604e-2$
 $\text{difference} = 5.12129896417792e-2$

x

$n = 10000$, left side = $1.38629438314442e-4$
 $1/n^{(3/10)} = 6.30957344480193e-2$
 $\text{difference} = 6.29571050097049e-2$
 $n = 20000$, left side = $6.93147191572763e-5$
 $1/n^{(3/10)} = 5.12496615052604e-2$
 $\text{difference} = 5.11803467861031e-2$

x^2

$n = 10000$, left side = $1.38684888448870e-4$
 $1/n^{(3/10)} = 6.30957344480193e-2$
 $\text{difference} = 6.29570495595705e-2$
 $n = 20000$, left side = $6.93285816908418e-5$
 $1/n^{(3/10)} = 5.12496615052604e-2$
 $\text{difference} = 5.11803329235695e-2$

x^3

$n = 10000$, left side = $1.04055271670012e-4$
 $1/n^{(3/10)} = 6.30957344480193e-2$
 $\text{difference} = 6.29916791763493e-2$
 $n = 20000$, left side = $5.20068353849767e-5$
 $1/n^{(3/10)} = 5.12496615052604e-2$
 $\text{difference} = 5.11976546698754e-2$

x^4

$n = 10000$, left side = $6.93979298337505e-5$
 $1/n^{(3/10)} = 6.30957344480193e-2$
 $\text{difference} = 6.30263365181856e-2$
 $n = 20000$, left side = $3.46781578120514e-5$

1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12149833474483e-2

$$x^{10}$$

n = 10000 , left side = 2.71736996781133e-6
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30930170780515e-2
 n = 20000 , left side = 1.35624196163052e-6
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12483052632987e-2

 x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1/2

$$x^{\frac{1}{3}}$$

n = 10000 , left side = 3.66622776546421e-5
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30590721703647e-2
 n = 20000 , left side = 1.83347566427017e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12313267486177e-2

$$x$$

n = 10000 , left side = 6.93147149063433e-5
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30264197331130e-2
 n = 20000 , left side = 3.46573574532272e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12150041478071e-2

$$x^2$$

```

n = 10000 , left side = 6.93557514500065e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30263786965693e-2
n = 20000 , left side = 3.46676165891013e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12149938886713e-2

```

$$x^3$$

```

n = 10000 , left side = 5.20475988625424e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30436868491568e-2
n = 20000 , left side = 2.60084077771394e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12236530974832e-2

```

$$x^4$$

```

n = 10000 , left side = 3.47189280094640e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30610155200099e-2
n = 20000 , left side = 1.73440694011678e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12323174358592e-2

```

$$x^{10}$$

```

n = 10000 , left side = 1.36102387619818e-6
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30943734241431e-2
n = 20000 , left side = 6.78705799766121e-7
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12489827994606e-2

```

x0 = 1/2, Power = 3/10, lamda = 1/2, q = 1

$$x^{\frac{1}{3}}$$

n = 10000 , left side = 1.27810567684961e-8
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30957216669626e-2
n = 20000 , left side = 3.19526349823462e-9
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12496583099969e-2

$$x$$

n = 10000 , left side = 0.000000000000000e0
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30957344480193e-2
n = 20000 , left side = 0.000000000000000e0
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12496615052604e-2

$$x^2$$

n = 10000 , left side = 3.62320168312280e-8
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30956982160025e-2
n = 20000 , left side = 9.05800418005143e-9
1/n^(3/10) = 5.12496615052604e-2
difference = 5.12496524472562e-2

$$x^3$$

n = 10000 , left side = 5.43480251358197e-8
1/n^(3/10) = 6.30957344480193e-2
difference = 6.30956800999942e-2
n = 20000 , left side = 1.35870062423216e-8

1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12496479182541e-2

$$x^4$$

n = 10000 , left side = 5.43480303677457e-8
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30956800999890e-2
 n = 20000 , left side = 1.35870062700771e-8
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12496479182541e-2

$$x^{10}$$

n = 10000 , left side = 6.36892661591386e-9
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30957280790927e-2
 n = 20000 , left side = 1.59222780251288e-9
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12496599130326e-2

 x0 = 1/2, Power = 3/10, lamda = 1, q = 1/4

$$x^{\frac{1}{3}}$$

n = 10000 , left side = 3.66548739493266e-5
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30590795740700e-2
 n = 20000 , left side = 1.83288788805358e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12313326263798e-2

$$x$$

```

n = 10000 , left side = 6.92842720413367e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30264501759780e-2
n = 20000 , left side = 3.46421360207794e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12150193692396e-2

```

$$x^2$$

```

n = 10000 , left side = 6.93006267839524e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30264338212354e-2
n = 20000 , left side = 3.46462247063362e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12150152805540e-2

```

$$x^3$$

```

n = 10000 , left side = 5.19877388821810e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30437467091371e-2
n = 20000 , left side = 2.59877353861737e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12236737698742e-2

```

$$x^4$$

```

n = 10000 , left side = 3.46666736098927e-5
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30610677744094e-2
n = 20000 , left side = 1.73272017113674e-5
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12323343035490e-2

```

$$x^{10}$$

```

n = 10000 , left side = 1.35608586396773e-6
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30943783621554e-2
n = 20000 , left side = 6.77323254739525e-7
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12489841820056e-2

```

x0 = 1/2, Power = 3/10, lamda = 1, q = 1/2

$$x^{\frac{1}{3}}$$

```

n = 10000 , left side = 1.83480025741645e-5
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30773864454452e-2
n = 20000 , left side = 9.17512596976255e-6
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12404863792906e-2

```

$$x$$

```

n = 10000 , left side = 3.46840545251137e-5
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30610503934942e-2
n = 20000 , left side = 1.73420272625568e-5
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12323194779978e-2

```

$$x^2$$

```

n = 10000 , left side = 3.46968096895939e-5
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30610376383297e-2
n = 20000 , left side = 1.73452160537879e-5

```


1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12323162892066e-2

$$x^3$$

n = 10000 , left side = 2.60321748816461e-5
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30697022731377e-2
 n = 20000 , left side = 1.30113037888624e-5
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12366502014715e-2

$$x^4$$

n = 10000 , left side = 1.73611624921383e-5
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30783732855272e-2
 n = 20000 , left side = 8.67579712438005e-6
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12409857081360e-2

$$x^{10}$$

n = 10000 , left side = 6.79666223719162e-7
 1/n^(3/10) = 6.30957344480193e-2
 difference = 6.30950547817956e-2
 n = 20000 , left side = 3.39272144287165e-7
 1/n^(3/10) = 5.12496615052604e-2
 difference = 5.12493222331161e-2

 x0 = 1/2, Power = 3/10, lamda = 1, q = 1

$$x^{\frac{1}{3}}$$

```

n = 10000 , left side = 4.08075506719285e-9
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30957303672643e-2
n = 20000 , left side = 1.02018837822016e-9
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12496604850720e-2

```

x

```

n = 10000 , left side = 0.000000000000000e0
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30957344480193e-2
n = 20000 , left side = 0.000000000000000e0
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12496615052604e-2

```

x^2

```

n = 10000 , left side = 1.15682139423257e-8
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30957228798054e-2
n = 20000 , left side = 2.89205359660372e-9
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12496586132068e-2

```

x^3

```

n = 10000 , left side = 1.73523209134885e-8
           1/n^(3/10) = 6.30957344480193e-2
           difference = 6.30957170956984e-2
n = 20000 , left side = 4.33808025612770e-9
           1/n^(3/10) = 5.12496615052604e-2
           difference = 5.12496571671801e-2

```

x^4

```

n = 10000 , left side = 1.73523213436999e-8
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30957170956980e-2
n = 20000 , left side = 4.33808787503320e-9
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12496571671725e-2

```

$$x^{10}$$

```

n = 10000 , left side = 2.03347633935362e-9
          1/n^(3/10) = 6.30957344480193e-2
          difference = 6.30957324145430e-2
n = 20000 , left side = 5.08369847954104e-10
          1/n^(3/10) = 5.12496615052604e-2
          difference = 5.12496609968905e-2

```

x0 = 1/2, Power = 1/2, lamda = 1/4, q = 1/4

$$x^{\frac{1}{3}}$$

```

n = 10000 , left side = 1.46632346878395e-4
          1/n^(1/2) = 1.00000000000000e-2
          difference = 9.85336765312161e-3
n = 20000 , left side = 7.33348323127547e-5
          1/n^(1/2) = 7.07106781186548e-3
          difference = 6.99773297955272e-3

```

$$x$$

```

n = 10000 , left side = 2.77258872223962e-4
          1/n^(1/2) = 1.00000000000000e-2
          difference = 9.72274112777604e-3
n = 20000 , left side = 1.38629436111981e-4

```

$1/n^{(1/2)} = 7.07106781186548e-3$
 difference = 6.93243837575349e-3

$$x^2$$

n = 10000 , left side = 2.77470672764790e-4
 $1/n^{(1/2)} = 1.00000000000000e-2$
 difference = 9.72252932723521e-3
 n = 20000 , left side = 1.38682386247202e-4
 $1/n^{(1/2)} = 7.07106781186548e-3$
 difference = 6.93238542561827e-3

$$x^3$$

n = 10000 , left side = 2.08261988522945e-4
 $1/n^{(1/2)} = 1.00000000000000e-2$
 difference = 9.79173801147705e-3
 n = 20000 , left side = 1.04051518979742e-4
 $1/n^{(1/2)} = 7.07106781186548e-3$
 difference = 6.96701629288573e-3

$$x^4$$

n = 10000 , left side = 1.38947404154099e-4
 $1/n^{(1/2)} = 1.00000000000000e-2$
 difference = 9.86105259584590e-3
 n = 20000 , left side = 6.93941766531508e-5
 $1/n^{(1/2)} = 7.07106781186548e-3$
 difference = 7.00167363521232e-3

$$x^{10}$$

n = 10000 , left side = 5.45256858194539e-6
 $1/n^{(1/2)} = 1.00000000000000e-2$
 difference = 9.99454743141806e-3
 n = 20000 , left side = 2.71692949521066e-6
 $1/n^{(1/2)} = 7.07106781186548e-3$

```
difference = 7.06835088237026e-3
```

6 Real-valued neural network approximation based on the q -deformed and λ -parametrized Hyperbolic Tangent - in the C# programming language

We have tested some of the results given above using C# (instead of SageMath).

It is important to note that these results have a limited accuracy. In general, “Double precision numbers are accurate up to sixteen decimal places but after calculations have been done there may be some rounding errors to account for.” [4]

We also created versions of our code that works with Decimal numbers - which allows computations with more significant decimal places.

In some cases we got the same results as seen in the sections above where we used SageMath, while in others, we got NaN (Not a number) or even “Unhandled exception. System.OverflowException: Value was either too large or too small for a Decimal.”

Please note that, as with the previous chapters, we removed several of the results generated by the code below.

```
[ ]: using System;
using System.Diagnostics;

StreamWriter outFile = new StreamWriter("Results.txt");

double[] powers = { 3.0 / 10, 1.0 / 2, 7.0 / 10 };
double[] lamdas = { 1.0 / 4, 1.0 / 2, 1 };
double[] qs = { 1.0 / 4, 1.0 / 2, 1 }; //deformation coefficient

//funcs = [x^(1/3), x] #choice of functions
List <Func < double, double > > funcs = new(); List funcNames
    => new();
```

```

funcNames.Add("x"); funcs.Add((x) => x); //for only a single
↳ statement
funcNames.Add("x^2"); funcs.Add((x)=> x * x); //for only a
↳ single statement
funcNames.Add("x^3"); funcs.Add((x) => x * x * x); //for only
↳ a single statement
funcNames.Add("x^4"); funcs.Add((x)=> x * x * x * x); //for
↳ only a single statement
funcNames.Add("x^10"); funcs.Add((x) => Math.Pow(x, 10)); //
↳ for only a single statement

double a = -1; // #the interval
double b = 1; // #the interval
double x0 = 1.0 / 2;

foreach (var power in powers) {
    foreach (var lamda in lamdas) {
        foreach (var q in qs) {
            outFile.WriteLine(); outFile.WriteLine();
            outFile.
↳ WriteLine("-----");
↳
            outFile.WriteLine($"x0 = {x0}, Power = {power} ,
↳ lamda = {lamda} , q = {q}");
            outFile.
↳ WriteLine("-----");
↳

            //the activation function
            Func<double, double, double, double> phi = (lamda, q,
↳ x) =>
            {
                return (Math.Pow(Math.E, (lamda * x)) - q * Math.
↳ Pow(Math.E, (-lamda * x)))

```

```

        / (Math.Pow(Math.E, (lamda * x)) + q * Math.
↪ Pow(Math.E, (-lamda * x))); // #formula 18.1
    };

    Func<double, double, double, double> G = (lamda, q, ↪
↪ x) =>
    {
        return 1.0 / 4 * (phi(lamda, q, x + 1) - ↪
↪ phi(lamda, q, x - 1)); // #formula 18.91
    };

    int indexF = -1;
    //
↪ #####
    foreach (var f in funcs)
    //
↪ #####
    {

        indexF++;

        //show(f(x))
        outFile.WriteLine($"{funcNames[indexF]}");
        foreach (var n in new int[] { 10, 20, 50, 100, ↪
↪ 200, 500, 1000, 2000, 5000, 10_000 }) //, 50_000, 100_000
        {
            Func<double, Func<double, double>, double, ↪
↪ double> L = (n, f, x) => // #real-valued linear neural ↪
↪ network operators
            {
                double totalNumer = 0;
                double totalDenom = 0;

```



```

        for (double k = Math.Ceiling(n * a); k
↪ <= Math.Floor(n * b); k++)
        {
            totalNumer += f(k / n) * G(lamda,
↪ q, n * x - k);
            totalDenom += G(lamda, q, n * x -
↪ k);
        }
        return totalNumer / totalDenom;
    };
    double computedL = L(n, f, x0);
    double computedf = f(x0);

    var leftSide = Math.Abs(computedL - computedf);
    outFile.WriteLine($" -----> L(n, f, x0) =
↪ {computedL}");
    outFile.WriteLine($" -----> f(x0) =
↪ {computedf}");
    outFile.WriteLine($" -----> leftSide =
↪ {leftSide}");
    var val1 = n;
    var val2 = leftSide;
    var val3 = 1 / (Math.Pow(n, power));
    outFile.WriteLine($"          n = {val1}, left
↪ side = {val2:F15}, 1 / n ^ {power:F4} = {val3:F15}");
    outFile.WriteLine($"          difference =
↪ {val3 - val2:F15} ");
    }
    }
    }
}
outFile.Close();

```

Here are some of the results obtained using the code above:

x0 = 0.5, Power = 0.3 , lamda = 0.25 , q = 0.25

x

```
-----> L(n, f, x0) = 0.6469605415088784
-----> f(x0) = 0.5
-----> leftSide = 0.14696054150887838
      n = 10, left side = 0.146960541508878, 1 / n^0.3000 = 0.
      ↪501187233627272
      difference = 0.354226692118394
```

```
-----> L(n, f, x0) = 0.6280448189448932
-----> f(x0) = 0.5
-----> leftSide = 0.12804481894489317
      n = 20, left side = 0.128044818944893, 1 / n^0.3000 = 0.
      ↪407090531536904
      difference = 0.279045712592011
```

```
-----> L(n, f, x0) = 0.5554458428059437
-----> f(x0) = 0.5
-----> leftSide = 0.05544584280594367
      n = 50, left side = 0.055445842805944, 1 / n^0.3000 = 0.
      ↪309249494710992
      difference = 0.253803651905048
```

```
-----> L(n, f, x0) = 0.5277258872001901
-----> f(x0) = 0.5
-----> leftSide = 0.02772588720019009
      n = 100, left side = 0.027725887200190, 1 / n^0.3000 = 0.
      ↪251188643150958
      difference = 0.223462755950768
```

```

-----> L(n, f, x0) = 0.513862943611199
-----> f(x0) = 0.5
-----> leftSide = 0.013862943611198997
      n = 200, left side = 0.013862943611199, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.190165633725638

```

```

-----> L(n, f, x0) = 0.5055451774444791
-----> f(x0) = 0.5
-----> leftSide = 0.005545177444479132
      n = 500, left side = 0.005545177444479, 1 / n^0.3000 = 0.
↪154991898754834
      difference = 0.149446721310355

```

```

-----> L(n, f, x0) = 0.5027725887222397
-----> f(x0) = 0.5
-----> leftSide = 0.0027725887222397327
      n = 1000, left side = 0.002772588722240, 1 / n^0.3000 = 0.
↪0.125892541179417
      difference = 0.123119952457177

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.5
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.5
-----> leftSide = NaN

```

```

n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.5
-----> leftSide = NaN
n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
difference = NaN

x^2
-----> L(n, f, x0) = 0.49406895721509386
-----> f(x0) = 0.25
-----> leftSide = 0.24406895721509386
n = 10, left side = 0.244068957215094, 1 / n^0.3000 = 0.
↪501187233627272
difference = 0.257118276412178

-----> L(n, f, x0) = 0.42339349636223705
-----> f(x0) = 0.25
-----> leftSide = 0.17339349636223705
n = 20, left side = 0.173393496362237, 1 / n^0.3000 = 0.
↪407090531536904
difference = 0.233697035174667

-----> L(n, f, x0) = 0.31391431405764025
-----> f(x0) = 0.25
-----> leftSide = 0.06391431405764025
n = 50, left side = 0.063914314057640, 1 / n^0.3000 = 0.
↪309249494710992
difference = 0.245335180653351

```

```

-----> L(n, f, x0) = 0.2798438925970292
-----> f(x0) = 0.25
-----> leftSide = 0.029843892597029187
      n = 100, left side = 0.029843892597029, 1 / n^0.3000 = 0.
      ↪ 251188643150958
      difference = 0.221344750553929

-----> L(n, f, x0) = 0.26439244496346914
-----> f(x0) = 0.25
-----> leftSide = 0.014392444963469142
      n = 200, left side = 0.014392444963469, 1 / n^0.3000 = 0.
      ↪ 204028577336837
      difference = 0.189636132373368

-----> L(n, f, x0) = 0.25562989766084293
-----> f(x0) = 0.25
-----> leftSide = 0.005629897660842931
      n = 500, left side = 0.005629897660843, 1 / n^0.3000 = 0.
      ↪ 154991898754834
      difference = 0.149362001093991

-----> L(n, f, x0) = 0.2527937687763307
-----> f(x0) = 0.25
-----> leftSide = 0.002793768776330696
      n = 1000, left side = 0.002793768776331, 1 / n^0.3000 = 0.
      ↪ 0.125892541179417
      difference = 0.123098772403086

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.25

```

```

-----> leftSide = NaN
      n = 2000, left side = NaN,  1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.25
-----> leftSide = NaN
      n = 5000, left side = NaN,  1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.25
-----> leftSide = NaN
      n = 10000, left side = NaN,  1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN

```

```

x^3
-----> L(n, f, x0) = 0.39553208148695584
-----> f(x0) = 0.125
-----> leftSide = 0.27053208148695584
      n = 10, left side = 0.270532081486956,  1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.230655152140317

```

```

-----> L(n, f, x0) = 0.3003038582607329
-----> f(x0) = 0.125
-----> leftSide = 0.17530385826073291
      n = 20, left side = 0.175303858260733,  1 / n^0.3000 = 0.
↪407090531536904

```

```
difference = 0.231786673276172
```

```
-----> L(n, f, x0) = 0.18035341668819518
```

```
-----> f(x0) = 0.125
```

```
-----> leftSide = 0.055353416688195184
```

```
      n = 50, left side = 0.055353416688195, 1 / n^0.3000 = 0.  
→309249494710992
```

```
difference = 0.253896078022797
```

```
-----> L(n, f, x0) = 0.14910496707077794
```

```
-----> f(x0) = 0.125
```

```
-----> leftSide = 0.024104967070777944
```

```
      n = 100, left side = 0.024104967070778, 1 / n^0.3000 = 0.  
→251188643150958
```

```
difference = 0.227083676080180
```

```
-----> L(n, f, x0) = 0.13620815268453845
```

```
-----> f(x0) = 0.125
```

```
-----> leftSide = 0.011208152684538453
```

```
      n = 200, left side = 0.011208152684538, 1 / n^0.3000 = 0.  
→204028577336837
```

```
difference = 0.192820424652299
```

```
-----> L(n, f, x0) = 0.1292870317565595
```

```
-----> f(x0) = 0.125
```

```
-----> leftSide = 0.004287031756559506
```

```
      n = 500, left side = 0.004287031756560, 1 / n^0.3000 = 0.  
→154991898754834
```

```
difference = 0.150704866998274
```

```
-----> L(n, f, x0) = 0.12711134516639788
```

```

-----> f(x0) = 0.125
-----> leftSide = 0.002111345166397882
      n = 1000, left side = 0.002111345166398, 1 / n^0.3000 = 0.125892541179417
      difference = 0.123781196013019

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.125
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.102256518256357
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.125
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.077679960971573
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.125
-----> leftSide = NaN
      n = 10000, left side = NaN, 1 / n^0.3000 = 0.063095734448019
      difference = NaN

```

```

x^4
-----> L(n, f, x0) = 0.33431165532114465
-----> f(x0) = 0.0625
-----> leftSide = 0.27181165532114465

```



```

n = 10, left side = 0.271811655321145, 1 / n^0.3000 = 0.
↪501187233627272
difference = 0.229375578306128

-----> L(n, f, x0) = 0.22220078089792553
-----> f(x0) = 0.0625
-----> leftSide = 0.15970078089792553
n = 20, left side = 0.159700780897926, 1 / n^0.3000 = 0.
↪407090531536904
difference = 0.247389750638979

-----> L(n, f, x0) = 0.10528678938945549
-----> f(x0) = 0.0625
-----> leftSide = 0.04278678938945549
n = 50, left side = 0.042786789389455, 1 / n^0.3000 = 0.
↪309249494710992
difference = 0.266462705321536

-----> L(n, f, x0) = 0.07982139153498581
-----> f(x0) = 0.0625
-----> leftSide = 0.01732139153498581
n = 100, left side = 0.017321391534986, 1 / n^0.3000 = 0.
↪251188643150958
difference = 0.233867251615972

-----> L(n, f, x0) = 0.07026000677274202
-----> f(x0) = 0.0625
-----> leftSide = 0.007760006772742023
n = 200, left side = 0.007760006772742, 1 / n^0.3000 = 0.
↪204028577336837
difference = 0.196268570564095

```

```

-----> L(n, f, x0) = 0.06540182870840235
-----> f(x0) = 0.0625
-----> leftSide = 0.002901828708402346
      n = 500, left side = 0.002901828708402, 1 / n^0.3000 = 0.
↪154991898754834
      difference = 0.152090070046431

```

```

-----> L(n, f, x0) = 0.06391833296468906
-----> f(x0) = 0.0625
-----> leftSide = 0.0014183329646890591
      n = 1000, left side = 0.001418332964689, 1 / n^0.3000 = 0.
↪0.125892541179417
      difference = 0.124474208214728

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0625
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0625
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0625
-----> leftSide = NaN

```

```

n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
difference = NaN

x^10
-----> L(n, f, x0) = 0.18782738583155176
-----> f(x0) = 0.0009765625
-----> leftSide = 0.18685082333155176
      n = 10, left side = 0.186850823331552, 1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.314336410295721

-----> L(n, f, x0) = 0.06681134142050742
-----> f(x0) = 0.0009765625
-----> leftSide = 0.06583477892050742
      n = 20, left side = 0.065834778920507, 1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.341255752616397

-----> L(n, f, x0) = 0.0058474132218746255
-----> f(x0) = 0.0009765625
-----> leftSide = 0.0048708507218746255
      n = 50, left side = 0.004870850721875, 1 / n^0.3000 = 0.
↪309249494710992
      difference = 0.304378643989117

-----> L(n, f, x0) = 0.0020762699143100087
-----> f(x0) = 0.0009765625
-----> leftSide = 0.0010997074143100087
      n = 100, left side = 0.001099707414310, 1 / n^0.3000 = 0.
↪251188643150958
      difference = 0.250088935736648

```

```

-----> L(n, f, x0) = 0.0013593673938147267
-----> f(x0) = 0.0009765625
-----> leftSide = 0.0003828048938147267
      n = 200, left side = 0.000382804893815, 1 / n^0.3000 = 0.
      ↪ 204028577336837
      difference = 0.203645772443022

-----> L(n, f, x0) = 0.0011008394681483109
-----> f(x0) = 0.0009765625
-----> leftSide = 0.00012427696814831086
      n = 500, left side = 0.000124276968148, 1 / n^0.3000 = 0.
      ↪ 154991898754834
      difference = 0.154867621786685

-----> L(n, f, x0) = 0.0010345676952653602
-----> f(x0) = 0.0009765625
-----> leftSide = 5.800519526536017E-05
      n = 1000, left side = 0.000058005195265, 1 / n^0.3000 = 0.
      ↪ 0.125892541179417
      difference = 0.125834535984151

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0009765625
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
      ↪ 102256518256357
      difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0009765625

```

```

-----> leftSide = NaN
      n = 5000, left side = NaN,  1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0009765625
-----> leftSide = NaN
      n = 10000, left side = NaN,  1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN

```

```

-----
x0 = 0.5, Power = 0.3 , lamda = 0.25 , q = 0.5
-----

```

```

x
-----> L(n, f, x0) = 0.5568097172543792
-----> f(x0) = 0.5
-----> leftSide = 0.05680971725437922
      n = 10, left side = 0.056809717254379,  1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.444377516372893

```

```

-----> L(n, f, x0) = 0.5632752819000139
-----> f(x0) = 0.5
-----> leftSide = 0.06327528190001386
      n = 20, left side = 0.063275281900014,  1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.343815249636891

```

```

-----> L(n, f, x0) = 0.5277227554082918

```

```

-----> f(x0) = 0.5
-----> leftSide = 0.027722755408291833
      n = 50, left side = 0.027722755408292, 1 / n^0.3000 = 0.
↪309249494710992
      difference = 0.281526739302700

-----> L(n, f, x0) = 0.5138629435997856
-----> f(x0) = 0.5
-----> leftSide = 0.013862943599785571
      n = 100, left side = 0.013862943599786, 1 / n^0.3000 = 0.
↪251188643150958
      difference = 0.237325699551172

-----> L(n, f, x0) = 0.5069314718055993
-----> f(x0) = 0.5
-----> leftSide = 0.006931471805599276
      n = 200, left side = 0.006931471805599, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.197097105531238

-----> L(n, f, x0) = 0.5027725887222398
-----> f(x0) = 0.5
-----> leftSide = 0.0027725887222398438
      n = 500, left side = 0.002772588722240, 1 / n^0.3000 = 0.
↪154991898754834
      difference = 0.152219310032594

-----> L(n, f, x0) = 0.5013862943611196
-----> f(x0) = 0.5
-----> leftSide = 0.0013862943611195888
      n = 1000, left side = 0.001386294361120, 1 / n^0.3000 = 0.
↪0.125892541179417

```

```
difference = 0.124506246818297
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.5
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.5
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.5
-----> leftSide = NaN
      n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN
```

x^2

```
-----> L(n, f, x0) = 0.39787275241087827
-----> f(x0) = 0.25
-----> leftSide = 0.14787275241087827
      n = 10, left side = 0.147872752410878, 1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.353314481216394
```

```

-----> L(n, f, x0) = 0.3478605569036921
-----> f(x0) = 0.25
-----> leftSide = 0.09786055690369211
      n = 20, left side = 0.097860556903692, 1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.309229974633212

-----> L(n, f, x0) = 0.2838868135786706
-----> f(x0) = 0.25
-----> leftSide = 0.03388681357867063
      n = 50, left side = 0.033886813578671, 1 / n^0.3000 = 0.
↪309249494710992
      difference = 0.275362681132321

-----> L(n, f, x0) = 0.26540440538603105
-----> f(x0) = 0.25
-----> leftSide = 0.01540440538603105
      n = 100, left side = 0.015404405386031, 1 / n^0.3000 = 0.
↪251188643150958
      difference = 0.235784237764927

-----> L(n, f, x0) = 0.25731683725369436
-----> f(x0) = 0.25
-----> leftSide = 0.007316837253694364
      n = 200, left side = 0.007316837253694, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.196711740083143

-----> L(n, f, x0) = 0.2528342471939351
-----> f(x0) = 0.25
-----> leftSide = 0.002834247193935102

```



```

n = 500, left side = 0.002834247193935, 1 / n^0.3000 = 0.
↪154991898754834
difference = 0.152157651560899

-----> L(n, f, x0) = 0.2514017089790437
-----> f(x0) = 0.25
-----> leftSide = 0.001401708979043681
n = 1000, left side = 0.001401708979044, 1 / n^0.3000 = ↪
↪0.125892541179417
difference = 0.124490832200373

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.25
-----> leftSide = NaN
n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.25
-----> leftSide = NaN
n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.25
-----> leftSide = NaN
n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
difference = NaN

```

x^3

```
-----> L(n, f, x0) = 0.29819200043142563
-----> f(x0) = 0.125
-----> leftSide = 0.17319200043142563
      n = 10, left side = 0.173192000431426, 1 / n^0.3000 = 0.
      ↪ 501187233627272
      difference = 0.327995233195847

-----> L(n, f, x0) = 0.22886492552153206
-----> f(x0) = 0.125
-----> leftSide = 0.10386492552153206
      n = 20, left side = 0.103864925521532, 1 / n^0.3000 = 0.
      ↪ 407090531536904
      difference = 0.303225606015372

-----> L(n, f, x0) = 0.15550737335115655
-----> f(x0) = 0.125
-----> leftSide = 0.030507373351156553
      n = 50, left side = 0.030507373351157, 1 / n^0.3000 = 0.
      ↪ 309249494710992
      difference = 0.278742121359835

-----> L(n, f, x0) = 0.13776817957524634
-----> f(x0) = 0.125
-----> leftSide = 0.012768179575246341
      n = 100, left side = 0.012768179575246, 1 / n^0.3000 = 0.
      ↪ 251188643150958
      difference = 0.238420463575712

-----> L(n, f, x0) = 0.1307839994262529
-----> f(x0) = 0.125
```

```

-----> leftSide = 0.005783999426252906
      n = 200, left side = 0.005783999426253, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.198244577910584

-----> L(n, f, x0) = 0.12717239948281703
-----> f(x0) = 0.125
-----> leftSide = 0.002172399482817028
      n = 500, left side = 0.002172399482817, 1 / n^0.3000 = 0.
↪154991898754834
      difference = 0.152819499272017

-----> L(n, f, x0) = 0.12606290147692495
-----> f(x0) = 0.125
-----> leftSide = 0.001062901476924949
      n = 1000, left side = 0.001062901476925, 1 / n^0.3000 = 0.
↪0.125892541179417
      difference = 0.124829639702492

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.125
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.125
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.125
-----> leftSide = NaN
      n = 10000, left side = NaN,  1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN

```

```

x^4
-----> L(n, f, x0) = 0.24275735469621276
-----> f(x0) = 0.0625
-----> leftSide = 0.18025735469621276
      n = 10, left side = 0.180257354696213,  1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.320929878931060

```

```

-----> L(n, f, x0) = 0.15884980900644186
-----> f(x0) = 0.0625
-----> leftSide = 0.09634980900644186
      n = 20, left side = 0.096349809006442,  1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.310740722530463

```

```

-----> L(n, f, x0) = 0.0866914336230854
-----> f(x0) = 0.0625
-----> leftSide = 0.0241914336230854
      n = 50, left side = 0.024191433623085,  1 / n^0.3000 = 0.
↪309249494710992
      difference = 0.285058061087906

```

```

-----> L(n, f, x0) = 0.07187035404303936

```

```

-----> f(x0) = 0.0625
-----> leftSide = 0.009370354043039361
      n = 100, left side = 0.009370354043039, 1 / n^0.3000 = 0.
↪251188643150958
      difference = 0.241818289107919

-----> L(n, f, x0) = 0.06655904957310244
-----> f(x0) = 0.0625
-----> leftSide = 0.004059049573102436
      n = 200, left side = 0.004059049573102, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.199969527763735

-----> L(n, f, x0) = 0.06397973714572872
-----> f(x0) = 0.0625
-----> leftSide = 0.0014797371457287156
      n = 500, left side = 0.001479737145729, 1 / n^0.3000 = 0.
↪154991898754834
      difference = 0.153512161609105

-----> L(n, f, x0) = 0.06321638757896159
-----> f(x0) = 0.0625
-----> leftSide = 0.0007163875789615859
      n = 1000, left side = 0.000716387578962, 1 / n^0.3000 = ↪
↪0.125892541179417
      difference = 0.125176153600455

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0625
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357

```

```
difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0625
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0625
-----> leftSide = NaN
      n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN
```

x^{10}

```
-----> L(n, f, x0) = 0.12023726345333331
-----> f(x0) = 0.0009765625
-----> leftSide = 0.11926070095333331
      n = 10, left side = 0.119260700953333, 1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.381926532673939
```

```
-----> L(n, f, x0) = 0.03832308332375248
-----> f(x0) = 0.0009765625
-----> leftSide = 0.03734652082375248
      n = 20, left side = 0.037346520823752, 1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.369744010713152
```

```

-----> L(n, f, x0) = 0.003778192061572009
-----> f(x0) = 0.0009765625
-----> leftSide = 0.002801629561572009
      n = 50, left side = 0.002801629561572, 1 / n^0.3000 = 0.
      ↪ 309249494710992
      difference = 0.306447865149420

-----> L(n, f, x0) = 0.0016094677211458996
-----> f(x0) = 0.0009765625
-----> leftSide = 0.0006329052211458996
      n = 100, left side = 0.000632905221146, 1 / n^0.3000 = 0.
      ↪ 251188643150958
      difference = 0.250555737929812

-----> L(n, f, x0) = 0.0011886064533363808
-----> f(x0) = 0.0009765625
-----> leftSide = 0.00021204395333638084
      n = 200, left side = 0.000212043953336, 1 / n^0.3000 = 0.
      ↪ 204028577336837
      difference = 0.203816533383501

-----> L(n, f, x0) = 0.0010420433182459323
-----> f(x0) = 0.0009765625
-----> leftSide = 6.54808182459323E-05
      n = 500, left side = 0.000065480818246, 1 / n^0.3000 = 0.
      ↪ 154991898754834
      difference = 0.154926417936588

-----> L(n, f, x0) = 0.00100640630978359
-----> f(x0) = 0.0009765625
-----> leftSide = 2.9843809783589893E-05

```

```

    n = 1000, left side = 0.000029843809784, 1 / n^0.3000 = 0.125892541179417
    difference = 0.125862697369633

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0009765625
-----> leftSide = NaN
    n = 2000, left side = NaN, 1 / n^0.3000 = 0.102256518256357
    difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0009765625
-----> leftSide = NaN
    n = 5000, left side = NaN, 1 / n^0.3000 = 0.077679960971573
    difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.0009765625
-----> leftSide = NaN
    n = 10000, left side = NaN, 1 / n^0.3000 = 0.063095734448019
    difference = NaN

```

Next, we use the cos and sin functions (we also changed the interval, and tried various values for x0). Here is the code:

```

[ ]:
using System;
using System.Diagnostics;

StreamWriter outFile = new StreamWriter("Results2.txt");

```



```

double[] powers = { 3.0 / 10, 1.0 / 2, 7.0 / 10 };
double[] lamdas = { 1.0 / 4, 1.0 / 2, 1 };
double[] qs = { 1.0 / 4, 1.0 / 2, 1 }; //deformation coefficient

//funcs = [x^(1/3), x] #choice of functions
List <Func < double, double > > funcs = new(); List funcNames
    = new();

funcNames.Add("sin(x)"); funcs.Add((x) => Math.Sin(x));
funcNames.Add("cos(x)"); funcs.Add((x) => Math.Cos(x));

double a = -Math.PI; // #the interval
double b = Math.PI; // #the interval
double[] x0s = { Math.PI / 4, Math.PI / 2, 3 * Math.PI / 4 };

foreach (var x0 in x0s) {
    foreach (var power in powers) {
        foreach (var lamda in lamdas) {
            foreach (var q in qs) {
                outFile.WriteLine(); outFile.WriteLine();
                outFile.
    WriteLine("-----");
    outFile.WriteLine($"x0 = {x0}, Power = {power} ,
    lamda = {lamda} , q = {q}");
                outFile.
    WriteLine("-----");

                //the activation function
                Func<double, double, double, double> phi = (lamda,
    q, x) =>

```

```

        {
            // #formula 18.1
            return (Math.Pow(Math.E, (lamda * x)) - q *
↪Math.Pow(Math.E, (-lamda * x)))
                / (Math.Pow(Math.E, (lamda * x)) + q * Math.
↪Pow(Math.E, (-lamda * x)));
        };

        Func<double, double, double, double> G = (lamda,
↪q, x) =>
        {
            // #formula 18.91
            return 1.0 / 4 * (phi(lamda, q, x + 1) -
↪phi(lamda, q, x - 1));
        };

        int indexF = -1;
        //
↪#####
        foreach (var f in funcs)
            //
↪#####
        {

            indexF++;

            //show(f(x))
            outFile.WriteLine($"{funcNames[indexF]}");
            foreach (var n in new int[] { 10, 20, 50, 100,
↪200, 500, 1000, 2000, 5000, 10_000 }) //, 50_000, 100_000
            {
                // #real-valued linear neural network
↪operators

```

```

        Func<double, Func<double, double>, double,
↳double> L = (n, f, x) =>
        {
            double totalNumer = 0;
            double totalDenom = 0;
            for (double k = Math.Ceiling(n * a); k
↳<= Math.Floor(n * b); k++)
            {
                totalNumer += f(k / n) * G(lamda,
↳q, n * x - k);
                totalDenom += G(lamda, q, n * x -
↳k);
            }
            return totalNumer / totalDenom;
        };
        double computedL = L(n, f, x0);
        double computedf = f(x0);

        var leftSide = Math.Abs(computedL -
↳computedf);

        outFile.WriteLine($" -----> L(n, f, x0)
↳= {computedL}");
        outFile.WriteLine($" -----> f(x0) =
↳{computedf}");
        outFile.WriteLine($" -----> leftSide =
↳{leftSide}");

        var val1 = n;
        var val2 = leftSide;
        var val3 = 1 / (Math.Pow(n, power));
        outFile.WriteLine($"          n = {val1},
↳left side = {val2:F15}, 1 / n ^ {power:F4} = {val3:F15}");
        outFile.WriteLine($"          difference =
↳{val3 - val2:F15} ");
    }
}

```

```

    }
  }
}
outFile.Close();

```

Here are some of the results obtained using the code above:

```

-----
x0 = 0.7853981633974483, Power = 0.3 , lamda = 0.25 , q = 0.25
-----

```

sin(x)

```

-----> L(n, f, x0) = 0.8173761586432317
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.11026937745668408
      n = 10, left side = 0.110269377456684, 1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.390917856170588

```

```

-----> L(n, f, x0) = 0.7847305388603272
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.0776237576737796
      n = 20, left side = 0.077623757673780, 1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.329466773863125

```

```

-----> L(n, f, x0) = 0.7432028965478586
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.036096115361311076

```

```

n = 50, left side = 0.036096115361311, 1 / n^0.3000 = 0.
↪309249494710992
difference = 0.273153379349681

-----> L(n, f, x0) = 0.7259478084258294
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.018841027239281827
n = 100, left side = 0.018841027239282, 1 / n^0.3000 = 0.
↪251188643150958
difference = 0.232347615911676

-----> L(n, f, x0) = 0.7167202150148566
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.009613433828309037
n = 200, left side = 0.009613433828309, 1 / n^0.3000 = 0.
↪204028577336837
difference = 0.194415143508528

-----> L(n, f, x0) = 0.710997735413702
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.003890954227154464
n = 500, left side = 0.003890954227154, 1 / n^0.3000 = 0.
↪154991898754834
difference = 0.151100944527679

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
n = 1000, left side = NaN, 1 / n^0.3000 = 0.
↪125892541179417
difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN

```

cos(x)

```

-----> L(n, f, x0) = 0.45523737617970633
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.25186940500684124
      n = 10, left side = 0.251869405006841, 1 / n^0.3000 = 0.
↪501187233627272
      difference = 0.249317828620431

```

```

-----> L(n, f, x0) = 0.5925643243319401

```

```

-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.11454245685460751
      n = 20, left side = 0.114542456854608, 1 / n^0.3000 = 0.
↪407090531536904
      difference = 0.292548074682297

-----> L(n, f, x0) = 0.6650335493966716
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.04207323178987599
      n = 50, left side = 0.042073231789876, 1 / n^0.3000 = 0.
↪309249494710992
      difference = 0.267176262921116

-----> L(n, f, x0) = 0.6867689433198332
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.020337837866714392
      n = 100, left side = 0.020337837866714, 1 / n^0.3000 = 0.
↪251188643150958
      difference = 0.230850805284244

-----> L(n, f, x0) = 0.6971189862142333
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.009987794972314279
      n = 200, left side = 0.009987794972314, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.194040782364523

-----> L(n, f, x0) = 0.7031559220730595
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.0039508591134880255
      n = 500, left side = 0.003950859113488, 1 / n^0.3000 = 0.
↪154991898754834

```

```
difference = 0.151041039641346
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 1000, left side = NaN, 1 / n^0.3000 = 0.
↪125892541179417
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN
```

$x_0 = 0.7853981633974483$, Power = 0.3 , lamda = 0.25 , q = 0.5

sin(x)

```
-----> L(n, f, x0) = 0.7466168412004801
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.039510060013932535
      n = 10, left side = 0.039510060013933, 1 / n^0.3000 = 0.
      ↪ 501187233627272
      difference = 0.461677173613340
```

```
-----> L(n, f, x0) = 0.7418056154262195
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.034698834239671905
      n = 20, left side = 0.034698834239672, 1 / n^0.3000 = 0.
      ↪ 407090531536904
      difference = 0.372391697297233
```

```
-----> L(n, f, x0) = 0.7244809726831161
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.017374191496568492
      n = 50, left side = 0.017374191496568, 1 / n^0.3000 = 0.
      ↪ 309249494710992
      difference = 0.291875303214423
```

```
-----> L(n, f, x0) = 0.7163577185903721
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.009250937403824544
      n = 100, left side = 0.009250937403825, 1 / n^0.3000 = 0.
      ↪ 251188643150958
      difference = 0.241937705747133
```

```

-----> L(n, f, x0) = 0.7118709756592027
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.004764194472655081
      n = 200, left side = 0.004764194472655, 1 / n^0.3000 = 0.
↪204028577336837
      difference = 0.199264382864182

```

```

-----> L(n, f, x0) = 0.7090454429256919
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.001938661739144365
      n = 500, left side = 0.001938661739144, 1 / n^0.3000 = 0.
↪154991898754834
      difference = 0.153053237015689

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 1000, left side = NaN, 1 / n^0.3000 = 0.
↪125892541179417
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

```

```

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN

```

```

    n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
    difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
    n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
    difference = NaN

cos(x)

-----> L(n, f, x0) = 0.5637961576061379
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.1433106235804097
    n = 10, left side = 0.143310623580410, 1 / n^0.3000 = 0.
↪501187233627272
    difference = 0.357876610046863

-----> L(n, f, x0) = 0.6454912281086387
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.06161555307790889
    n = 20, left side = 0.061615553077909, 1 / n^0.3000 = 0.
↪407090531536904
    difference = 0.345474978458996

-----> L(n, f, x0) = 0.6853812716148499
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.02172550957169772
    n = 50, left side = 0.021725509571698, 1 / n^0.3000 = 0.
↪309249494710992

```

```
difference = 0.287523985139294
```

```
-----> L(n, f, x0) = 0.6967664035262482
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.010340377660299382
      n = 100, left side = 0.010340377660299, 1 / n^0.3000 = 0.
      ↪ 251188643150958
      difference = 0.240848265490659
```

```
-----> L(n, f, x0) = 0.7020701258174417
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.0050366553691059135
      n = 200, left side = 0.005036655369106, 1 / n^0.3000 = 0.
      ↪ 204028577336837
      difference = 0.198991921967731
```

```
-----> L(n, f, x0) = 0.7051245211848308
-----> f(x0) = 0.7071067811865476
-----> leftSide = 0.0019822600017167513
      n = 500, left side = 0.001982260001717, 1 / n^0.3000 = 0.
      ↪ 154991898754834
      difference = 0.153009638753117
```

```
-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 1000, left side = NaN, 1 / n^0.3000 = 0.
      ↪ 125892541179417
      difference = NaN
```

```
-----> L(n, f, x0) = NaN
```

```

-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 2000, left side = NaN, 1 / n^0.3000 = 0.
↪102256518256357
      difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 5000, left side = NaN, 1 / n^0.3000 = 0.
↪077679960971573
      difference = NaN

-----> L(n, f, x0) = NaN
-----> f(x0) = 0.7071067811865476
-----> leftSide = NaN
      n = 10000, left side = NaN, 1 / n^0.3000 = 0.
↪063095734448019
      difference = NaN

```

7 Bivariate case approximation

7.1 Introduction

We present in here some of the background and the main result that was proven in the monograph [1], in Chapter 17, named "Banach Space Valued Multivariate Multi Layer Neural Network Approximation Based on q -Deformed and λ -Parametrized Hyperbolic Tangent Function".

The **activation function** [see monograph [1], formula 17.1] used for this part is defined as follows:

$$g_{q,\lambda}(x) := \frac{e^{\lambda x} - qe^{-\lambda x}}{e^{\lambda x} + qe^{-\lambda x}}, \forall x \in \mathbb{R}, \text{ where } q, \lambda > 0. \quad (5)$$

Then [see [1], formulae 17.9 and 17.58], we present the **density function**:

$$M_{q,\lambda}(x) := \frac{1}{4}(g_{q,\lambda}(x+1) - qg_{q,\lambda}(x-1)) > 0, \forall x \in \mathbb{R}, \text{ where } q, \lambda > 0, \quad (6)$$

and for the bivariate operator:

$$Z_{q,\lambda}(x_1, x_2) := M_{q,\lambda}(x_1) \cdot M_{q,\lambda}(x_2), \forall x = (x_1, x_2) \in \mathbb{R}^2, \text{ where } q, \lambda > 0. \quad (7)$$

Lastly, (see [1], formula 17.69), we give the real-valued **bivariate linear normalized neural network operator**:

$$A_n(f, x_1, x_2) := \frac{\sum_{k_1=\lceil na_1 \rceil}^{\lfloor nb_1 \rfloor} \sum_{k_2=\lceil na_2 \rceil}^{\lfloor nb_2 \rfloor} f\left(\frac{k_1}{n}, \frac{k_2}{n}\right) (M_{q,\lambda}(nx_1 - k_1) \cdot M_{q,\lambda}(nx_2 - k_2))}{\left(\sum_{k_1=\lceil na_1 \rceil}^{\lfloor nb_1 \rfloor} M_{q,\lambda}(nx_1 - k_1)\right) \cdot \left(\sum_{k_2=\lceil na_2 \rceil}^{\lfloor nb_2 \rfloor} M_{q,\lambda}(nx_2 - k_2)\right)}, \quad (8)$$

where $f \in C([a_1, b_1] \times [a_2, b_2])$, $x_i \in [a_i, b_i]$, $i = 1, 2$, $q, \lambda > 0$.

It was shown (see [1], Theorem 17.9), that:

$$\lim_{n \rightarrow \infty} A_n(f) = f, \quad (9)$$

pointwise and uniformly.

Next, we present our computational results using C#.

7.2 Using C#

[]:

```
using System;
using System.Diagnostics;

StreamWriter outFile = new StreamWriter("ResultsMultivariate.
    ↪txt");

double[] powers = { 3.0 / 10 }; // { 3.0 / 10, 1.0 / 2, 7.0 / ↪
    ↪10 };
double[] lamdas = { 1.0 / 4, 1.0 / 2, 1 };
double[] qs = { 1.0 / 4, 1.0 / 2 }; //deformation coefficient

//funcs = [x^(1/3), x] #choice of functions
List<Func<double, double, double>> funcs = new();
List<string> funcNames = new();

funcNames.Add("sin(x*y)"); funcs.Add((x,y) => Math.Sin(x*y)); /
    ↪/for only a single statement
funcNames.Add("cos(x+y)"); funcs.Add((x,y) => Math.Cos(x+y)); /
    ↪/for only a single statement

double a = -Math.PI; // #the interval
double b = Math.PI; // #the interval
double[] x0s = { Math.PI / 4, Math.PI / 2, 3 * Math.PI / 4 };
double[] y0s = { Math.PI / 4, Math.PI / 2, 3 * Math.PI / 4 };

//the activation function
Func<double, double, double, double> phi = (lamda, q, x) =>
{
    return (Math.Pow(Math.E, (lamda * x)) - q * Math.Pow(Math.E, ↪
    ↪(-lamda * x)))
```



```

    / (Math.Pow(Math.E, (lamda * x)) + q * Math.Pow(Math.E,
    ↪(-lamda * x))); // #formula 17.1
};

foreach (var x0 in x0s)
{
    foreach (var y0 in y0s)
    {
        foreach (var power in powers)
        {
            foreach (var lamda in lamdas)
            {
                foreach (var q in qs)
                {
                    outFile.WriteLine();
                    outFile.WriteLine();
                    outFile.
    ↪WriteLine("-----");
    ↪
                    outFile.WriteLine($"x0 = {x0}, y0={y0}, Power =
    ↪{power} , lamda = {lamda} , q = {q}");
                    outFile.
    ↪WriteLine("-----");
    ↪

                    Func<double, double, double, double> G = (lamda, q,
    ↪x) =>
                    {
                        return 1.0 / 4 * (phi(lamda, q, x + 1) -
    ↪phi(lamda, q, x - 1)); //see formula 17.9
                    };

```

```

        int indexF = -1;
        //
        #####
        foreach (var f in funcs)
            //
            #####
            {

                indexF++;

                //show(f(x))
                outFile.WriteLine($"{funcNames[indexF]}");
                foreach (var n in new int[] { 1, 5, 10, 100, 200,
500}) //see formula 17.69
                {

                    double totalNumer = 0;

                    double sum1 = 0, sum2 = 0;
                    for (double k1 = Math.Ceiling(n * a); k1 <=
Math.Floor(n * b); k1++)
                    {
                        for (double k2 = Math.Ceiling(n * a); k2 <=
Math.Floor(n * b); k2++)
                        {
                            totalNumer += f(k1 / n, k2/n) * G(lamda,
q, n * x0 - k1) * G(lamda, q, n * y0 - k2);
                        }
                    }

                    for (double k = Math.Ceiling(n * a); k <= Math.
Floor(n * b); k++)

```



```
x0=0.7853981633974483, y0=0.7853981633974483, Power=0.3,
↳lamda=0.25, q=0.25
```

```
-----
sin(x*y)
```

```
-----> n = 1, leftSide = 0.5603358492312364, difference = 0.
↳439664150768764
```

```
-----> n = 5, leftSide = 0.1310742319590139, difference = 0.
↳485959630760996
```

```
-----> n = 10, leftSide = 0.18848685417437927, difference =
↳0.312700379452893
```

```
-----> n = 100, leftSide = 0.03502349900488344, difference
↳= 0.216165144146075
```

```
-----> n = 200, leftSide = 0.017649853643137314, difference
↳= 0.186378723693700
```

```
-----> n = 500, leftSide = 0.00708818592339322, difference
↳= 0.147903712831440
```

```
cos(x+y)
```

```
-----> n = 1, leftSide = 0.0505057384628108, difference = 0.
↳949494261537189
```

```
-----> n = 5, leftSide = 0.5508946280390616, difference = 0.
↳066139234680948
```

```
-----> n = 10, leftSide = 0.46086271604738127, difference =
↳0.040324517579891
```

```
-----> n = 100, leftSide = 0.05534863904962483, difference
↳= 0.195840004101333
```

```
-----> n = 200, leftSide = 0.02771298567058189, difference
↳= 0.176315591666255
```

```
-----> n = 500, leftSide = 0.011089529016998289, difference
↳= 0.143902369737835
```

```
-----
x0=0.7853981633974483, y0=0.7853981633974483, Power=0.3,
↳lamda=0.25, q=0.5
-----
```

```

sin(x*y)
-----> n = 1, leftSide = 0.568511828999265, difference = 0.
↪431488171000735
-----> n = 5, leftSide = 0.1177810989519571, difference = 0.
↪499252763768052
-----> n = 10, leftSide = 0.08163830326496868, difference =
↪0.419548930362304
-----> n = 100, leftSide = 0.01726327023878793, difference
↪= 0.233925372912170
-----> n = 200, leftSide = 0.008761299593207594, difference
↪= 0.195267277743629
-----> n = 500, leftSide = 0.0035337716671465236,
↪difference = 0.151458127087687
cos(x+y)
-----> n = 1, leftSide = 0.03321770228863349, difference =
↪0.966782297711366
-----> n = 5, leftSide = 0.31725762812120784, difference =
↪0.299776234598802
-----> n = 10, leftSide = 0.23957060023273813, difference =
↪0.261616633394534
-----> n = 100, leftSide = 0.027684959901099458, difference
↪= 0.223503683249859
-----> n = 200, leftSide = 0.013857824420667101, difference
↪= 0.190170752916170
-----> n = 500, leftSide = 0.005544849757553625, difference
↪= 0.149447048997280

```

```

-----
x0=0.7853981633974483, y0=0.7853981633974483, Power=0.3,
↪lamda=0.5, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.5242063334974406, difference = 0.
↪475793666502559

```

```

-----> n = 5, leftSide = 0.17824399502907606, difference =
↪0.438789867690934
-----> n = 10, leftSide = 0.15128511297820368, difference =
↪0.349902120649069
-----> n = 100, leftSide = 0.017640319028731866, difference
↪= 0.233548324122226
-----> n = 200, leftSide = 0.008852350926071395, difference
↪= 0.195176226410766
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.17105176049078444, difference =
↪0.828948239509216
-----> n = 5, leftSide = 0.4562711432689492, difference = 0.
↪160762719451060
-----> n = 10, leftSide = 0.2640084241521494, difference =
↪0.237178809475123
-----> n = 100, leftSide = 0.027712292550153908, difference
↪= 0.223476350600804
-----> n = 200, leftSide = 0.013861244054517835, difference
↪= 0.190167333282319
-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=0.7853981633974483, y0=0.7853981633974483, Power=0.3,
↪lamda=0.5, q=0.5
-----
sin(x*y)
-----> n = 1, leftSide = 0.5271829720826484, difference = 0.
↪472817027917352
-----> n = 5, leftSide = 0.0741568440105349, difference = 0.
↪542877018709475
-----> n = 10, leftSide = 0.0703249708411523, difference =
↪0.430862262786120

```

```

-----> n = 100, leftSide = 0.008752069678607088, difference =
↪= 0.242436573472351
-----> n = 200, leftSide = 0.004408969337715174, difference =
↪= 0.199619607999122
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.12406518105940369, difference =
↪0.875934818940596
-----> n = 5, leftSide = 0.23717964442617406, difference =
↪0.379854218293836
-----> n = 10, leftSide = 0.13328288047686793, difference =
↪0.367904353150404
-----> n = 100, leftSide = 0.013857478646570491, difference =
↪= 0.237331164504388
-----> n = 200, leftSide = 0.0069307881539852284,
↪difference = 0.197097789182852
-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=0.7853981633974483, y0=0.7853981633974483, Power=0.3,
↪lamda=1, q=0.25
-----
sin(x*y)
-----> n = 1, leftSide = 0.3766446931098869, difference = 0.
↪623355306890113
-----> n = 5, leftSide = 0.14433319051535165, difference =
↪0.472700672204658
-----> n = 10, leftSide = 0.08306812279488662, difference =
↪0.418119110832386
-----> n = 100, leftSide = 0.008847268931951424, difference =
↪= 0.242341374219007
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)

```

```

-----> n = 1, leftSide = 0.3654430015411151, difference = 0.
↪634556998458885
-----> n = 5, leftSide = 0.2612978659627136, difference = 0.
↪355735996757296
-----> n = 10, leftSide = 0.13658111312006863, difference =
↪0.364606120507204
-----> n = 100, leftSide = 0.013867359594955293, difference
↪= 0.237321283556003
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=0.7853981633974483, y0=0.7853981633974483, Power=0.3,
↪lamda=1, q=0.5
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.37546113193021197, difference =
↪0.624538868069788
-----> n = 5, leftSide = 0.06500801161307324, difference =
↪0.552025851106936
-----> n = 10, leftSide = 0.0395742976828517, difference =
↪0.461612935944421
-----> n = 100, leftSide = 0.004397141443465813, difference
↪= 0.246791501707492
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

cos(x+y)

```

-----> n = 1, leftSide = 0.22408470026778576, difference =
↪0.775915299732214
-----> n = 5, leftSide = 0.132085066395304, difference = 0.
↪484948796324706
-----> n = 10, leftSide = 0.06852554674473353, difference =
↪0.432661686882539

```



```

-----> n = 100, leftSide = 0.0069263586404547444,
↪ difference = 0.244262284510503
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=0.7853981633974483, y0=1.5707963267948966, Power=0.3,
↪ lamda=0.25, q=0.25
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.9239545552925531, difference = 0.
↪ 076045444707447
-----> n = 5, leftSide = 0.7407181930290099, difference =
↪ -0.123684330309000
-----> n = 10, leftSide = 0.24846261949434578, difference =
↪ 0.252724614132927
-----> n = 100, leftSide = 0.017695212963459328, difference
↪ = 0.233493430187499
-----> n = 200, leftSide = 0.009848946875683828, difference
↪ = 0.194179630461153
-----> n = 500, leftSide = 0.004170939343787428, difference
↪ = 0.150820959411046

```

cos(x+y)

```

-----> n = 1, leftSide = 0.6563123309488232, difference = 0.
↪ 343687669051177
-----> n = 5, leftSide = 0.06595275106552934, difference =
↪ 0.551081111654480
-----> n = 10, leftSide = 0.14585722146238667, difference =
↪ 0.355330012164886
-----> n = 100, leftSide = 0.037098680879204315, difference
↪ = 0.214089962271754
-----> n = 200, leftSide = 0.019085890398383487, difference
↪ = 0.184942686938453

```

```

-----> n = 500, leftSide = 0.00775983612541431, difference
↪= 0.147232062629419

-----
x0=0.7853981633974483, y0=1.5707963267948966, Power=0.3,
↪lamda=0.25, q=0.5
-----

sin(x*y)
-----> n = 1, leftSide = 0.9315564478407371, difference = 0.
↪068443552159263
-----> n = 5, leftSide = 0.5556726989987174, difference = 0.
↪061361163721292
-----> n = 10, leftSide = 0.164787654867126, difference = 0.
↪336399578760146
-----> n = 100, leftSide = 0.008329407978986403, difference
↪= 0.242859235171972
-----> n = 200, leftSide = 0.004791944719788499, difference
↪= 0.199236632617048
-----> n = 500, leftSide = 0.0020639803683792923,
↪difference = 0.152927918386454

cos(x+y)
-----> n = 1, leftSide = 0.6674215419953312, difference = 0.
↪332578458004669
-----> n = 5, leftSide = 0.09746599838599257, difference =
↪0.519567864334017
-----> n = 10, leftSide = 0.057866594908942526, difference
↪= 0.443320638718330
-----> n = 100, leftSide = 0.01835150152180387, difference
↪= 0.232837141629154
-----> n = 200, leftSide = 0.009492565812904474, difference
↪= 0.194536011523932
-----> n = 500, leftSide = 0.003871767875614718, difference
↪= 0.151120130879219

```

x0=0.7853981633974483, y0=1.5707963267948966, Power=0.3, □

↪ lamda=0.5, q=0.25

sin(x*y)

-----> n = 1, leftSide = 0.8966568724411209, difference = 0.

↪ 103343127558879

-----> n = 5, leftSide = 0.26243205180466256, difference = □

↪ 0.354601810915347

-----> n = 10, leftSide = 0.009896258492545718, difference □

↪ = 0.491290975134727

-----> n = 100, leftSide = 0.009811385946394746, difference □

↪ = 0.241377257204563

-----> n = 200, leftSide = 0.005156820776599402, difference □

↪ = 0.198871756560238

-----> n = 500, leftSide = NaN, difference = NaN

cos(x+y)

-----> n = 1, leftSide = 0.5507682666492701, difference = 0.

↪ 449231733350730

-----> n = 5, leftSide = 0.13765334679964802, difference = □

↪ 0.479380515920362

-----> n = 10, leftSide = 0.1355469185117204, difference = □

↪ 0.365640315115552

-----> n = 100, leftSide = 0.019067735679156983, difference □

↪ = 0.232120907471801

-----> n = 200, leftSide = 0.009669394152941657, difference □

↪ = 0.194359183183895

-----> n = 500, leftSide = NaN, difference = NaN

x0=0.7853981633974483, y0=1.5707963267948966, Power=0.3, □

↪ lamda=0.5, q=0.5

```

sin(x*y)
-----> n = 1, leftSide = 0.8909282712186807, difference = 0.
↪109071728781319
-----> n = 5, leftSide = 0.1794802023467934, difference = 0.
↪437553660373216
-----> n = 10, leftSide = 0.01759699920183666, difference =
↪0.483590234425436
-----> n = 100, leftSide = 0.004754973527030915, difference
↪= 0.246433669623927
-----> n = 200, leftSide = 0.002540359332032449, difference
↪= 0.201488218004805
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.5438131333830379, difference = 0.
↪456186866616962
-----> n = 5, leftSide = 0.05032774694127595, difference =
↪0.566706115778734
-----> n = 10, leftSide = 0.06261350361998574, difference =
↪0.438573730007287
-----> n = 100, leftSide = 0.009474650977698063, difference
↪= 0.241713992173260
-----> n = 200, leftSide = 0.0048197759685643105,
↪difference = 0.199208801368273
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=0.7853981633974483, y0=1.5707963267948966, Power=0.3,
↪lamda=1, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.821907473372728, difference = 0.
↪178092526627272
-----> n = 5, leftSide = 0.027646303919920734, difference =
↪0.589387558800089

```

```

-----> n = 10, leftSide = 0.022689484951259886, difference_
↪= 0.478497748676012
-----> n = 100, leftSide = 0.005120633165550226, difference_
↪= 0.246068009985408
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.324991077838988, difference = 0.
↪675008922161012
-----> n = 5, leftSide = 0.127146841158018, difference = 0.
↪489887021561992
-----> n = 10, leftSide = 0.08177513454609897, difference =_
↪0.419412099081173
-----> n = 100, leftSide = 0.00965147597778071, difference_
↪= 0.241537167173177
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=0.7853981633974483, y0=1.5707963267948966, Power=0.3,_
↪lamda=1, q=0.5
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.7673300548315893, difference = 0.
↪232669945168411
-----> n = 5, leftSide = 0.03381145165229105, difference =_
↪0.583222411067719
-----> n = 10, leftSide = 0.006051113924451013, difference_
↪= 0.495136119702821
-----> n = 100, leftSide = 0.0025029048556297617,_
↪difference = 0.248685738295328
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)

```

```

-----> n = 1, leftSide = 0.3285634867991426, difference = 0.
↪671436513200857
-----> n = 5, leftSide = 0.05501692071740949, difference =
↪0.562016942002600
-----> n = 10, leftSide = 0.03863376377363903, difference =
↪0.462553469853633
-----> n = 100, leftSide = 0.004801509423281836, difference
↪= 0.246387133727676
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=0.7853981633974483, y0=2.356194490192345, Power=0.3, lamda=0.
↪25, q=0.25
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.9402161080978435, difference = 0.
↪059783891902156
-----> n = 5, leftSide = 0.9498934678762577, difference =
↪-0.332859605156248
-----> n = 10, leftSide = 0.7017348119261979, difference =
↪-0.200547578298926
-----> n = 100, leftSide = 0.031922814262192345, difference
↪= 0.219265828888766
-----> n = 200, leftSide = 0.013993822680222756, difference
↪= 0.190034754656614
-----> n = 500, leftSide = 0.0051248395052390094,
↪difference = 0.149867059249595

```

cos(x+y)

```

-----> n = 1, leftSide = 0.9503357585315514, difference = 0.
↪049664241468449
-----> n = 5, leftSide = 0.4537739831199443, difference = 0.
↪163259879600065

```

```

-----> n = 10, leftSide = 0.2184911945049388, difference = 0.282696039122334
-----> n = 100, leftSide = 0.0028831814041029302, difference = 0.248305461746855
-----> n = 200, leftSide = 0.0007214606192176287, difference = 0.203307116717619
-----> n = 500, leftSide = 0.00011546352666280679, difference = 0.154876435228171

```

```

-----
x0=0.7853981633974483, y0=2.356194490192345, Power=0.3, lamda=0.25, q=0.5
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.9474589716636656, difference = 0.052541028336334
-----> n = 5, leftSide = 0.7602911061398281, difference = -0.143257243419818
-----> n = 10, leftSide = 0.4540208519979142, difference = 0.047166381629358
-----> n = 100, leftSide = 0.016984461765722658, difference = 0.234204181385235
-----> n = 200, leftSide = 0.007254166382035354, difference = 0.196774410954802
-----> n = 500, leftSide = 0.0026037301881448593, difference = 0.152388168566689

```

cos(x+y)

```

-----> n = 1, leftSide = 0.9563509123366939, difference = 0.043649087663306
-----> n = 5, leftSide = 0.34562263752865396, difference = 0.271411225191356
-----> n = 10, leftSide = 0.14037856419010586, difference = 0.360808669437166

```

```

-----> n = 100, leftSide = 0.001732017559051724, difference
↪= 0.249456625591906
-----> n = 200, leftSide = 0.00043330910632766173,
↪difference = 0.203595268230509
-----> n = 500, leftSide = 6.934311749240418E-05,
↪difference = 0.154922555637341

-----
x0=0.7853981633974483, y0=2.356194490192345, Power=0.3, lamda=0.
↪5, q=0.25
-----

sin(x*y)
-----> n = 1, leftSide = 0.9202607864783152, difference = 0.
↪079739213521685
-----> n = 5, leftSide = 0.6974652378150932, difference =
↪-0.080431375095084
-----> n = 10, leftSide = 0.3046267525099131, difference =
↪0.196560481117359
-----> n = 100, leftSide = 0.014067884449064816, difference
↪= 0.237120758701893
-----> n = 200, leftSide = 0.006523008571408639, difference
↪= 0.197505568765428
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.8688161767510898, difference = 0.
↪131183823248910
-----> n = 5, leftSide = 0.2155924409832557, difference = 0.
↪401441421736754
-----> n = 10, leftSide = 0.07161943645948254, difference =
↪0.429567797167790
-----> n = 100, leftSide = 0.0007464423906479167,
↪difference = 0.250442200760310
-----> n = 200, leftSide = 0.0001866556200013081,
↪difference = 0.203841921716836

```



```

-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=0.7853981633974483, y0=2.356194490192345, Power=0.3, lamda=0.
↪5, q=0.5
-----

sin(x*y)
-----> n = 1, leftSide = 0.9153890310526238, difference = 0.
↪084610968947376
-----> n = 5, leftSide = 0.46321818652950186, difference = ↪
↪0.153815676190508
-----> n = 10, leftSide = 0.18103979384090818, difference = ↪
↪0.320147439786364
-----> n = 100, leftSide = 0.007328233712312393, difference ↪
↪= 0.243860409438646
-----> n = 200, leftSide = 0.0033352748130388576, ↪
↪difference = 0.200693302523798
-----> n = 500, leftSide = NaN, difference = NaN

cos(x+y)
-----> n = 1, leftSide = 0.8225483073255397, difference = 0.
↪177451692674460
-----> n = 5, leftSide = 0.14378240587447833, difference = ↪
↪0.473251456845531
-----> n = 10, leftSide = 0.044409431173700664, difference ↪
↪= 0.456777802453572
-----> n = 100, leftSide = 0.0004582980713653262, ↪
↪difference = 0.250730345079593
-----> n = 200, leftSide = 0.00011459564480531537, ↪
↪difference = 0.203913981692032
-----> n = 500, leftSide = NaN, difference = NaN

-----

```

```
x0=0.7853981633974483, y0=2.356194490192345, Power=0.3,
↳lamda=1, q=0.25
```

```
-----
sin(x*y)
```

```
-----> n = 1, leftSide = 0.9105735246046178, difference = 0.
↳089426475395382
```

```
-----> n = 5, leftSide = 0.32557317589075074, difference =
↳0.291460686829259
```

```
-----> n = 10, leftSide = 0.1173377621393622, difference =
↳0.383849471487910
```

```
-----> n = 100, leftSide = 0.006600127404657874, difference
↳= 0.244588515746300
```

```
-----> n = 200, leftSide = NaN, difference = NaN
```

```
-----> n = 500, leftSide = NaN, difference = NaN
```

```
cos(x+y)
```

```
-----> n = 1, leftSide = 0.6790890976440375, difference = 0.
↳320910902355963
```

```
-----> n = 5, leftSide = 0.08005712680315735, difference =
↳0.536976735916852
```

```
-----> n = 10, leftSide = 0.020967288708739873, difference
↳= 0.480219944918532
```

```
-----> n = 100, leftSide = 0.0002117230559122163,
↳difference = 0.250976920095046
```

```
-----> n = 200, leftSide = NaN, difference = NaN
```

```
-----> n = 500, leftSide = NaN, difference = NaN
```

```
-----
x0=0.7853981633974483, y0=2.356194490192345, Power=0.3,
↳lamda=1, q=0.5
-----
```

```
sin(x*y)
```

```
-----> n = 1, leftSide = 0.8635798426028963, difference = 0.
↳136420157397104
```

```

-----> n = 5, leftSide = 0.20577724025342536, difference =
↪0.411256622466584
-----> n = 10, leftSide = 0.06958999856682024, difference =
↪0.431597235060452
-----> n = 100, leftSide = 0.003408025614788701, difference
↪= 0.247780617536169
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.5852072744273846, difference = 0.
↪414792725572615
-----> n = 5, leftSide = 0.05360161831086174, difference =
↪0.563432244409148
-----> n = 10, leftSide = 0.013866203183831916, difference
↪= 0.487321030443440
-----> n = 100, leftSide = 0.00013966152120425956,
↪difference = 0.251048981629754
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=1.5707963267948966, y0=0.7853981633974483, Power=0.3,
↪lamda=0.25, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.9239545552925531, difference = 0.
↪076045444707447
-----> n = 5, leftSide = 0.7407181930290097, difference =
↪-0.123684330309000
-----> n = 10, leftSide = 0.24846261949434556, difference =
↪0.252724614132927
-----> n = 100, leftSide = 0.01769521296345733, difference
↪= 0.233493430187501

```

```

-----> n = 200, leftSide = 0.009848946875685494, difference
↪= 0.194179630461151
-----> n = 500, leftSide = 0.0041709393437773246,
↪difference = 0.150820959411056
cos(x+y)
-----> n = 1, leftSide = 0.6563123309488232, difference = 0.
↪343687669051177
-----> n = 5, leftSide = 0.06595275106552956, difference =
↪0.551081111654480
-----> n = 10, leftSide = 0.14585722146238522, difference =
↪0.355330012164887
-----> n = 100, leftSide = 0.03709868087920154, difference
↪= 0.214089962271756
-----> n = 200, leftSide = 0.019085890398382377, difference
↪= 0.184942686938455
-----> n = 500, leftSide = 0.007759836125418973, difference
↪= 0.147232062629415

```

```

x0=1.5707963267948966, y0=0.7853981633974483, Power=0.3,
↪lamda=0.25, q=0.5

```

```

sin(x*y)
-----> n = 1, leftSide = 0.9315564478407371, difference = 0.
↪068443552159263
-----> n = 5, leftSide = 0.5556726989987166, difference = 0.
↪061361163721293
-----> n = 10, leftSide = 0.1647876548671281, difference =
↪0.336399578760144
-----> n = 100, leftSide = 0.008329407978986292, difference
↪= 0.242859235171972
-----> n = 200, leftSide = 0.004791944719790608, difference
↪= 0.199236632617046

```

```

-----> n = 500, leftSide = 0.0020639803683659697,
↪difference = 0.152927918386468
cos(x+y)
-----> n = 1, leftSide = 0.6674215419953312, difference = 0.
↪332578458004669
-----> n = 5, leftSide = 0.09746599838599246, difference =
↪0.519567864334017
-----> n = 10, leftSide = 0.057866594908943525, difference
↪= 0.443320638718329
-----> n = 100, leftSide = 0.018351501521805202, difference
↪= 0.232837141629153
-----> n = 200, leftSide = 0.009492565812902365, difference
↪= 0.194536011523935
-----> n = 500, leftSide = 0.00387176787562149, difference
↪= 0.151120130879212

```

```

-----
x0=1.5707963267948966, y0=0.7853981633974483, Power=0.3,
↪lamda=0.5, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.8966568724411209, difference = 0.
↪103343127558879
-----> n = 5, leftSide = 0.26243205180466256, difference =
↪0.354601810915347
-----> n = 10, leftSide = 0.00989625849254605, difference =
↪0.491290975134726
-----> n = 100, leftSide = 0.00981138594639508, difference
↪= 0.241377257204563
-----> n = 200, leftSide = 0.005156820776599402, difference
↪= 0.198871756560238
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)

```

```

-----> n = 1, leftSide = 0.5507682666492703, difference = 0.
↪449231733350730
-----> n = 5, leftSide = 0.1376533467996477, difference = 0.
↪479380515920362
-----> n = 10, leftSide = 0.13554691851172052, difference =
↪0.365640315115552
-----> n = 100, leftSide = 0.01906773567915976, difference
↪= 0.232120907471798
-----> n = 200, leftSide = 0.009669394152939992, difference
↪= 0.194359183183897
-----> n = 500, leftSide = NaN, difference = NaN

```

```

x0=1.5707963267948966, y0=0.7853981633974483, Power=0.3,
↪lamda=0.5, q=0.5

```

sin(x*y)

```

-----> n = 1, leftSide = 0.8909282712186807, difference = 0.
↪109071728781319
-----> n = 5, leftSide = 0.1794802023467934, difference = 0.
↪437553660373216
-----> n = 10, leftSide = 0.01759699920183999, difference =
↪0.483590234425432
-----> n = 100, leftSide = 0.00475497352702936, difference
↪= 0.246433669623929
-----> n = 200, leftSide = 0.002540359332030895, difference
↪= 0.201488218004806
-----> n = 500, leftSide = NaN, difference = NaN

```

cos(x+y)

```

-----> n = 1, leftSide = 0.5438131333830378, difference = 0.
↪456186866616962
-----> n = 5, leftSide = 0.050327746941276064, difference =
↪0.566706115778734

```

```

-----> n = 10, leftSide = 0.06261350361998619, difference = 0.438573730007286
-----> n = 100, leftSide = 0.009474650977699284, difference = 0.241713992173259
-----> n = 200, leftSide = 0.004819775968562312, difference = 0.199208801368275
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=1.5707963267948966, y0=0.7853981633974483, Power=0.3, lamda=1, q=0.25
-----

```

```

sin(x*y)

```

```

-----> n = 1, leftSide = 0.8219074733727281, difference = 0.178092526627272
-----> n = 5, leftSide = 0.0276463039199214, difference = 0.589387558800088
-----> n = 10, leftSide = 0.02268948495125933, difference = 0.478497748676013
-----> n = 100, leftSide = 0.005120633165548227, difference = 0.246068009985410
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

cos(x+y)

```

```

-----> n = 1, leftSide = 0.324991077838988, difference = 0.675008922161012
-----> n = 5, leftSide = 0.127146841158017, difference = 0.489887021561993
-----> n = 10, leftSide = 0.0817751345460983, difference = 0.419412099081174
-----> n = 100, leftSide = 0.009651475977781154, difference = 0.241537167173177
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```
-----  
x0=1.5707963267948966, y0=0.7853981633974483, Power=0.3, □  
  ↪ lamda=1, q=0.5  
-----
```

```
sin(x*y)
```

```
-----> n = 1, leftSide = 0.7673300548315893, difference = 0.  
  ↪ 232669945168411  
-----> n = 5, leftSide = 0.03381145165229105, difference = □  
  ↪ 0.583222411067719  
-----> n = 10, leftSide = 0.006051113924450902, difference □  
  ↪ = 0.495136119702821  
-----> n = 100, leftSide = 0.0025029048556293176, □  
  ↪ difference = 0.248685738295329  
-----> n = 200, leftSide = NaN, difference = NaN  
-----> n = 500, leftSide = NaN, difference = NaN
```

```
cos(x+y)
```

```
-----> n = 1, leftSide = 0.32856348679914266, difference = □  
  ↪ 0.671436513200857  
-----> n = 5, leftSide = 0.05501692071740927, difference = □  
  ↪ 0.562016942002600  
-----> n = 10, leftSide = 0.03863376377363836, difference = □  
  ↪ 0.462553469853634  
-----> n = 100, leftSide = 0.004801509423281947, difference □  
  ↪ = 0.246387133727676  
-----> n = 200, leftSide = NaN, difference = NaN  
-----> n = 500, leftSide = NaN, difference = NaN
```

```
-----  
x0=1.5707963267948966, y0=1.5707963267948966, Power=0.3, □  
  ↪ lamda=0.25, q=0.25  
-----
```

```
sin(x*y)
```



```

-----> n = 1, leftSide = 0.6026147996589162, difference = 0.
↪397385200341084
-----> n = 5, leftSide = 0.7576994505433036, difference =
↪-0.140665587823294
-----> n = 10, leftSide = 0.7660185735493232, difference =
↪-0.264831339922051
-----> n = 100, leftSide = 0.07286744970092618, difference
↪= 0.178321193450032
-----> n = 200, leftSide = 0.03525843845438681, difference
↪= 0.168770138882450
-----> n = 500, leftSide = 0.013809505442841541, difference
↪= 0.141182393311992
cos(x+y)
-----> n = 1, leftSide = 0.9504584513751807, difference = 0.
↪049541548624819
-----> n = 5, leftSide = 0.577491592200249, difference = 0.
↪039542270519761
-----> n = 10, leftSide = 0.2522910132797712, difference =
↪0.248896220347501
-----> n = 100, leftSide = 0.0028831814041004877,
↪difference = 0.248305461746858
-----> n = 200, leftSide = 0.0007214606192106343,
↪difference = 0.203307116717626
-----> n = 500, leftSide = 0.00011546352664959514,
↪difference = 0.154876435228184

```

```

-----
x0=1.5707963267948966, y0=1.5707963267948966, Power=0.3,
↪lamda=0.25, q=0.5
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.6093663308625931, difference = 0.
↪390633669137407

```

```

-----> n = 5, leftSide = 0.5610247774776371, difference = 0.
↪056009085242372
-----> n = 10, leftSide = 0.4450144239704958, difference =
↪0.056172809656777
-----> n = 100, leftSide = 0.03674976150498799, difference
↪= 0.214438881645970
-----> n = 200, leftSide = 0.017705689943415703, difference
↪= 0.186322887393421
-----> n = 500, leftSide = 0.006916768653180472, difference
↪= 0.148075130101653
cos(x+y)
-----> n = 1, leftSide = 0.9540246654362885, difference = 0.
↪045975334563712
-----> n = 5, leftSide = 0.4064304049969918, difference = 0.
↪210603457723018
-----> n = 10, leftSide = 0.15639670379129256, difference =
↪0.344790529835980
-----> n = 100, leftSide = 0.0017320175590486153,
↪difference = 0.249456625591909
-----> n = 200, leftSide = 0.0004333091063186689,
↪difference = 0.203595268230518
-----> n = 500, leftSide = 6.9343117476528E-05, difference
↪= 0.154922555637357

-----
x0=1.5707963267948966, y0=1.5707963267948966, Power=0.3,
↪lamda=0.5, q=0.25
-----
sin(x*y)
-----> n = 1, leftSide = 0.5822952705583352, difference = 0.
↪417704729441665
-----> n = 5, leftSide = 0.7559909175736127, difference =
↪-0.138957054853603

```

```

-----> n = 10, leftSide = 0.42510422525089975, difference =
↪0.076083008376373
-----> n = 100, leftSide = 0.03529538460827253, difference
↪= 0.215893258542685
-----> n = 200, leftSide = 0.01733324606478437, difference
↪= 0.186695331272053
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.8927358523754148, difference = 0.
↪107264147624585
-----> n = 5, leftSide = 0.2584237378720501, difference = 0.
↪358610124847960
-----> n = 10, leftSide = 0.07231497281905863, difference =
↪0.428872260808214
-----> n = 100, leftSide = 0.0007464423578245061,
↪difference = 0.250442200793134
-----> n = 200, leftSide = 0.00018665561921904494,
↪difference = 0.203841921717618
-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=1.5707963267948966, y0=1.5707963267948966, Power=0.3,
↪lamda=0.5, q=0.5
-----
sin(x*y)
-----> n = 1, leftSide = 0.5712591065614122, difference = 0.
↪428740893438588
-----> n = 5, leftSide = 0.44508273517328206, difference =
↪0.171951127546728
-----> n = 10, leftSide = 0.22546562367032164, difference =
↪0.275721609956951
-----> n = 100, leftSide = 0.01774340568001498, difference
↪= 0.233445237470943

```

```

-----> n = 200, leftSide = 0.008690265698572675, difference_
↪= 0.195338311638264
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.8306217533788608, difference = 0.
↪169378246621139
-----> n = 5, leftSide = 0.1641617584594547, difference = 0.
↪452872104260555
-----> n = 10, leftSide = 0.04473340484449506, difference =_
↪0.456453828782777
-----> n = 100, leftSide = 0.0004582980591308905,_
↪difference = 0.250730345091827
-----> n = 200, leftSide = 0.0001145956449623009,_
↪difference = 0.203913981691875
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=1.5707963267948966, y0=1.5707963267948966, Power=0.3,_
↪lamda=1, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.5864627875423122, difference = 0.
↪413537212457688
-----> n = 5, leftSide = 0.4287981920363517, difference = 0.
↪188235670683658
-----> n = 10, leftSide = 0.20093969529286165, difference =_
↪0.300247538334411
-----> n = 100, leftSide = 0.017362961147294076, difference_
↪= 0.233825682003664
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.7775838073968335, difference = 0.
↪222416192603166

```

```

-----> n = 5, leftSide = 0.08151153166798486, difference =
↪0.535522331052025
-----> n = 10, leftSide = 0.020966437843750962, difference
↪= 0.480220795783521
-----> n = 100, leftSide = 0.00021157613089495264,
↪difference = 0.250977067020063
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=1.5707963267948966, y0=1.5707963267948966, Power=0.3,
↪lamda=1, q=0.5
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.5057556442116913, difference = 0.
↪494244355788309
-----> n = 5, leftSide = 0.2346664067784031, difference = 0.
↪382367455941606
-----> n = 10, leftSide = 0.10506887541473142, difference =
↪0.396118358212541
-----> n = 100, leftSide = 0.008731863296700282, difference
↪= 0.242456779854258
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

cos(x+y)

```

-----> n = 1, leftSide = 0.6436935676998752, difference = 0.
↪356306432300125
-----> n = 5, leftSide = 0.05428305599937022, difference =
↪0.562750806720639
-----> n = 10, leftSide = 0.013870693229541708, difference
↪= 0.487316540397731
-----> n = 100, leftSide = 0.00013952141038031662,
↪difference = 0.251049121740578
-----> n = 200, leftSide = NaN, difference = NaN

```

```

-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=1.5707963267948966, y0=2.356194490192345, Power=0.3, lamda=0.
  ↪25, q=0.25
-----

sin(x*y)
-----> n = 1, leftSide = 0.5537842093586645, difference = 0.
  ↪446215790641336
-----> n = 5, leftSide = 0.37551401483085034, difference = 0.
  ↪0.241519847889159
-----> n = 10, leftSide = 0.02663661968240949, difference = 0.
  ↪0.474550613944863
-----> n = 100, leftSide = 0.08611285934143897, difference = 0.
  ↪0.165075783809519
-----> n = 200, leftSide = 0.04469670818492533, difference = 0.
  ↪0.159331869151912
-----> n = 500, leftSide = 0.01823431892300409, difference = 0.
  ↪0.136757579831830

cos(x+y)
-----> n = 1, leftSide = 0.660152586826157, difference = 0.
  ↪339847413173843
-----> n = 5, leftSide = 0.5670364770160042, difference = 0.
  ↪049997385704005
-----> n = 10, leftSide = 0.4557508128202348, difference = 0.
  ↪0.045436420807038
-----> n = 100, leftSide = 0.04117611512366948, difference = 0.
  ↪0.210012528027289
-----> n = 200, leftSide = 0.020106189790788487, difference = 0.
  ↪0.183922387546048
-----> n = 500, leftSide = 0.0079231262107613, difference = 0.
  ↪0.147068772544072

```

x0=1.5707963267948966, y0=2.356194490192345, Power=0.3, lamda=0.
→25, q=0.5

sin(x*y)

-----> n = 1, leftSide = 0.547569451503161, difference = 0.
→452430548496839
-----> n = 5, leftSide = 0.38034214102619746, difference =
→0.236691721693812
-----> n = 10, leftSide = 0.032163269965970465, difference
→= 0.469023963661302
-----> n = 100, leftSide = 0.04233088568827148, difference
→= 0.208857757462687
-----> n = 200, leftSide = 0.022156076723690243, difference
→= 0.181872500613147
-----> n = 500, leftSide = 0.009085359449139019, difference
→= 0.145906539305695

cos(x+y)

-----> n = 1, leftSide = 0.6579976470662093, difference = 0.
→342002352933791
-----> n = 5, leftSide = 0.3377738443426027, difference = 0.
→279260018377407
-----> n = 10, leftSide = 0.2513911570566785, difference =
→0.249796076570594
-----> n = 100, leftSide = 0.02080094424408141, difference
→= 0.230387698906877
-----> n = 200, leftSide = 0.010105357427786177, difference
→= 0.193923219909051
-----> n = 500, leftSide = 0.003969833852825011, difference
→= 0.151022064902009

x0=1.5707963267948966, y0=2.356194490192345, Power=0.3, lamda=0.
→5, q=0.25

sin(x*y)

-----> n = 1, leftSide = 0.5704248425429528, difference = 0.
↪ 429575157457047

-----> n = 5, leftSide = 0.009919594222051353, difference =
↪ 0.607114268497958

-----> n = 10, leftSide = 0.23303120616958128, difference =
↪ 0.268156027457691

-----> n = 100, leftSide = 0.04463818187578661, difference
↪ = 0.206550461275171

-----> n = 200, leftSide = 0.022707338656893006, difference
↪ = 0.181321238679944

-----> n = 500, leftSide = NaN, difference = NaN

cos(x+y)

-----> n = 1, leftSide = 0.6517138357795285, difference = 0.
↪ 348286164220472

-----> n = 5, leftSide = 0.4417370424804969, difference = 0.
↪ 175296820239513

-----> n = 10, leftSide = 0.23666474907473672, difference =
↪ 0.264522484552536

-----> n = 100, leftSide = 0.02012336472332099, difference
↪ = 0.231065278427637

-----> n = 200, leftSide = 0.00993336495227648, difference
↪ = 0.194095212384560

-----> n = 500, leftSide = NaN, difference = NaN

x0=1.5707963267948966, y0=2.356194490192345, Power=0.3, lamda=0.
↪ 5, q=0.5

sin(x*y)

-----> n = 1, leftSide = 0.5764461054459599, difference = 0.
↪ 423553894554040


```

-----> n = 5, leftSide = 0.05496840388241481, difference =
↪0.562065458837595
-----> n = 10, leftSide = 0.10301135506292736, difference =
↪0.398175878564345
-----> n = 100, leftSide = 0.022100217951421697, difference
↪= 0.229088425199536
-----> n = 200, leftSide = 0.011297600056453083, difference
↪= 0.192730977280384
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.5588128792203702, difference = 0.
↪441187120779630
-----> n = 5, leftSide = 0.2459443806488938, difference = 0.
↪371089482071116
-----> n = 10, leftSide = 0.12526363870721335, difference =
↪0.375923594920059
-----> n = 100, leftSide = 0.010122782300449829, difference
↪= 0.241065860850508
-----> n = 200, leftSide = 0.004981838752162626, difference
↪= 0.199046738584674
-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=1.5707963267948966, y0=2.356194490192345, Power=0.3,
↪lamda=1, q=0.25
-----
sin(x*y)
-----> n = 1, leftSide = 0.5271717580108257, difference = 0.
↪472828241989174
-----> n = 5, leftSide = 0.19770756255080557, difference =
↪0.419326300169204
-----> n = 10, leftSide = 0.17545000849985937, difference =
↪0.325737225127413

```

```

-----> n = 100, leftSide = 0.022649081258080628, difference = 0.228539561892877
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.666766673600017, difference = 0.333233326399983
-----> n = 5, leftSide = 0.23998424116413997, difference = 0.377049621555870
-----> n = 10, leftSide = 0.11145546263263473, difference = 0.389731770994638
-----> n = 100, leftSide = 0.009950631514490715, difference = 0.241238011636467
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=1.5707963267948966, y0=2.356194490192345, Power=0.3, lamda=1, q=0.5
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.5653606011950689, difference = 0.434639398804931
-----> n = 5, leftSide = 0.07432229274307256, difference = 0.542711569976937
-----> n = 10, leftSide = 0.08060332438948326, difference = 0.420583909237789
-----> n = 100, leftSide = 0.01124510674757051, difference = 0.239943536403388
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.4778374955528845, difference = 0.522162504447115

```

```

-----> n = 5, leftSide = 0.13044635524604042, difference =
↪0.486587507473969
-----> n = 10, leftSide = 0.058211923670450805, difference
↪= 0.442975309956822
-----> n = 100, leftSide = 0.004999939310337598, difference
↪= 0.246188703840620
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=2.356194490192345, y0=0.7853981633974483, Power=0.3, lamda=0.
↪25, q=0.25
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.9402161080978435, difference = 0.
↪059783891902156
-----> n = 5, leftSide = 0.9498934678762575, difference =
↪-0.332859605156248
-----> n = 10, leftSide = 0.7017348119261975, difference =
↪-0.200547578298925
-----> n = 100, leftSide = 0.03192281426219079, difference
↪= 0.219265828888767
-----> n = 200, leftSide = 0.01399382268021876, difference
↪= 0.190034754656618
-----> n = 500, leftSide = 0.005124839505234791, difference
↪= 0.149867059249599

```

cos(x+y)

```

-----> n = 1, leftSide = 0.9503357585315514, difference = 0.
↪049664241468449
-----> n = 5, leftSide = 0.45377398311994455, difference =
↪0.163259879600065
-----> n = 10, leftSide = 0.21849119450493704, difference =
↪0.282696039122335

```

```

-----> n = 100, leftSide = 0.0028831814040992665,
↪difference = 0.248305461746859
-----> n = 200, leftSide = 0.0007214606192108564,
↪difference = 0.203307116717626
-----> n = 500, leftSide = 0.00011546352665803283,
↪difference = 0.154876435228176

```

```

-----
x0=2.356194490192345, y0=0.7853981633974483, Power=0.3, lamda=0.
↪25, q=0.5
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.9474589716636656, difference = 0.
↪052541028336334
-----> n = 5, leftSide = 0.7602911061398283, difference =
↪-0.143257243419819
-----> n = 10, leftSide = 0.45402085199791353, difference =
↪0.047166381629359
-----> n = 100, leftSide = 0.016984461765722547, difference
↪= 0.234204181385235
-----> n = 200, leftSide = 0.007254166382030469, difference
↪= 0.196774410954806
-----> n = 500, leftSide = 0.002603730188143638, difference
↪= 0.152388168566690

```

cos(x+y)

```

-----> n = 1, leftSide = 0.9563509123366939, difference = 0.
↪043649087663306
-----> n = 5, leftSide = 0.3456226375286541, difference = 0.
↪271411225191356
-----> n = 10, leftSide = 0.14037856419010464, difference =
↪0.360808669437168
-----> n = 100, leftSide = 0.0017320175590491704,
↪difference = 0.249456625591909

```

```

-----> n = 200, leftSide = 0.00043330910632521924,
↪difference = 0.203595268230512
-----> n = 500, leftSide = 6.934311748874045E-05,
↪difference = 0.154922555637345

-----
x0=2.356194490192345, y0=0.7853981633974483, Power=0.3, lamda=0.
↪5, q=0.25
-----

sin(x*y)
-----> n = 1, leftSide = 0.9202607864783152, difference = 0.
↪079739213521685
-----> n = 5, leftSide = 0.6974652378150936, difference =
↪-0.080431375095084
-----> n = 10, leftSide = 0.3046267525099131, difference =
↪0.196560481117359
-----> n = 100, leftSide = 0.0140678844490697, difference =
↪0.237120758701888
-----> n = 200, leftSide = 0.0065230085714090835,
↪difference = 0.197505568765428
-----> n = 500, leftSide = NaN, difference = NaN

cos(x+y)
-----> n = 1, leftSide = 0.8688161767510898, difference = 0.
↪131183823248910
-----> n = 5, leftSide = 0.21559244098325714, difference =
↪0.401441421736752
-----> n = 10, leftSide = 0.07161943645948099, difference =
↪0.429567797167791
-----> n = 100, leftSide = 0.0007464423906504702,
↪difference = 0.250442200760308
-----> n = 200, leftSide = 0.0001866556200018632,
↪difference = 0.203841921716835
-----> n = 500, leftSide = NaN, difference = NaN

```

```
-----  
x0=2.356194490192345, y0=0.7853981633974483, Power=0.3, lamda=0.  
  ↪5, q=0.5  
-----
```

```
sin(x*y)
```

```
-----> n = 1, leftSide = 0.9153890310526239, difference = 0.  
  ↪084610968947376  
-----> n = 5, leftSide = 0.46321818652950136, difference =  
  ↪0.153815676190508  
-----> n = 10, leftSide = 0.1810397938409093, difference =  
  ↪0.320147439786363  
-----> n = 100, leftSide = 0.007328233712313947, difference  
  ↪= 0.243860409438644  
-----> n = 200, leftSide = 0.0033352748130385246,  
  ↪difference = 0.200693302523798  
-----> n = 500, leftSide = NaN, difference = NaN
```

```
cos(x+y)
```

```
-----> n = 1, leftSide = 0.8225483073255397, difference = 0.  
  ↪177451692674460  
-----> n = 5, leftSide = 0.143782405874478, difference = 0.  
  ↪473251456845532  
-----> n = 10, leftSide = 0.04440943117370211, difference =  
  ↪0.456777802453570  
-----> n = 100, leftSide = 0.0004582980713698781,  
  ↪difference = 0.250730345079588  
-----> n = 200, leftSide = 0.0001145956448059815,  
  ↪difference = 0.203913981692031  
-----> n = 500, leftSide = NaN, difference = NaN
```

```
-----  
x0=2.356194490192345, y0=0.7853981633974483, Power=0.3,  
  ↪lamda=1, q=0.25  
-----
```

```

sin(x*y)
-----> n = 1, leftSide = 0.9105735246046179, difference = 0.
↪089426475395382
-----> n = 5, leftSide = 0.3255731758907503, difference = 0.
↪291460686829259
-----> n = 10, leftSide = 0.11733776213936231, difference =
↪0.383849471487910
-----> n = 100, leftSide = 0.006600127404657652, difference
↪= 0.244588515746300
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

cos(x+y)
-----> n = 1, leftSide = 0.6790890976440376, difference = 0.
↪320910902355962
-----> n = 5, leftSide = 0.08005712680315691, difference =
↪0.536976735916853
-----> n = 10, leftSide = 0.020967288708740317, difference
↪= 0.480219944918532
-----> n = 100, leftSide = 0.0002117230559112171,
↪difference = 0.250976920095047
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=2.356194490192345, y0=0.7853981633974483, Power=0.3,
↪lamda=1, q=0.5
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.8635798426028964, difference = 0.
↪136420157397104
-----> n = 5, leftSide = 0.20577724025342514, difference =
↪0.411256622466584
-----> n = 10, leftSide = 0.06958999856682002, difference =
↪0.431597235060452

```

```

-----> n = 100, leftSide = 0.00340802561478748, difference
↪= 0.247780617536171
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.5852072744273848, difference = 0.
↪414792725572615
-----> n = 5, leftSide = 0.053601618310860966, difference =
↪0.563432244409149
-----> n = 10, leftSide = 0.01386620318383236, difference =
↪0.487321030443440
-----> n = 100, leftSide = 0.00013966152120381548,
↪difference = 0.251048981629754
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=2.356194490192345, y0=1.5707963267948966, Power=0.3, lamda=0.
↪25, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.5537842093586645, difference = 0.
↪446215790641336
-----> n = 5, leftSide = 0.37551401483085045, difference =
↪0.241519847889159
-----> n = 10, leftSide = 0.026636619682409157, difference
↪= 0.474550613944863
-----> n = 100, leftSide = 0.08611285934143931, difference
↪= 0.165075783809519
-----> n = 200, leftSide = 0.04469670818492821, difference
↪= 0.159331869151909
-----> n = 500, leftSide = 0.01823431892300642, difference
↪= 0.136757579831827
cos(x+y)

```



```

-----> n = 1, leftSide = 0.6601525868261571, difference = 0.
↪339847413173843
-----> n = 5, leftSide = 0.5670364770160046, difference = 0.
↪049997385704005
-----> n = 10, leftSide = 0.4557508128202357, difference =
↪0.045436420807037
-----> n = 100, leftSide = 0.04117611512366748, difference
↪= 0.210012528027291
-----> n = 200, leftSide = 0.02010618979078682, difference
↪= 0.183922387546050
-----> n = 500, leftSide = 0.00792312621075797, difference
↪= 0.147068772544076

```

```

-----
x0=2.356194490192345, y0=1.5707963267948966, Power=0.3, lamda=0.
↪25, q=0.5
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.547569451503161, difference = 0.
↪452430548496839
-----> n = 5, leftSide = 0.38034214102619746, difference =
↪0.236691721693812
-----> n = 10, leftSide = 0.03216326996597102, difference =
↪0.469023963661301
-----> n = 100, leftSide = 0.04233088568827226, difference
↪= 0.208857757462686
-----> n = 200, leftSide = 0.02215607672369424, difference
↪= 0.181872500613143
-----> n = 500, leftSide = 0.009085359449142238, difference
↪= 0.145906539305691

```

cos(x+y)

```

-----> n = 1, leftSide = 0.6579976470662093, difference = 0.
↪342002352933791

```

```

-----> n = 5, leftSide = 0.33777384434260255, difference =
↪0.279260018377407
-----> n = 10, leftSide = 0.2513911570566783, difference =
↪0.249796076570594
-----> n = 100, leftSide = 0.020800944244082742, difference
↪= 0.230387698906875
-----> n = 200, leftSide = 0.01010535742778329, difference
↪= 0.193923219909054
-----> n = 500, leftSide = 0.0039698338528224575,
↪difference = 0.151022064902011

```

```

-----
x0=2.356194490192345, y0=1.5707963267948966, Power=0.3, lamda=0.
↪5, q=0.25
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.5704248425429528, difference = 0.
↪429575157457047
-----> n = 5, leftSide = 0.009919594222051464, difference =
↪0.607114268497958
-----> n = 10, leftSide = 0.23303120616958273, difference =
↪0.268156027457690
-----> n = 100, leftSide = 0.044638181875786276, difference
↪= 0.206550461275172
-----> n = 200, leftSide = 0.022707338656893117, difference
↪= 0.181321238679944
-----> n = 500, leftSide = NaN, difference = NaN

```

cos(x+y)

```

-----> n = 1, leftSide = 0.6517138357795285, difference = 0.
↪348286164220472
-----> n = 5, leftSide = 0.4417370424804969, difference = 0.
↪175296820239513
-----> n = 10, leftSide = 0.23666474907473656, difference =
↪0.264522484552536

```

```

-----> n = 100, leftSide = 0.020123364723321324, difference_
↪= 0.231065278427637
-----> n = 200, leftSide = 0.00993336495227759, difference_
↪= 0.194095212384559
-----> n = 500, leftSide = NaN, difference = NaN

-----
x0=2.356194490192345, y0=1.5707963267948966, Power=0.3, lamda=0.
↪5, q=0.5
-----

sin(x*y)
-----> n = 1, leftSide = 0.5764461054459599, difference = 0.
↪423553894554040
-----> n = 5, leftSide = 0.05496840388241503, difference =_
↪0.562065458837595
-----> n = 10, leftSide = 0.10301135506292924, difference =_
↪0.398175878564343
-----> n = 100, leftSide = 0.022100217951421697, difference_
↪= 0.229088425199536
-----> n = 200, leftSide = 0.011297600056453527, difference_
↪= 0.192730977280383
-----> n = 500, leftSide = NaN, difference = NaN

cos(x+y)
-----> n = 1, leftSide = 0.5588128792203702, difference = 0.
↪441187120779630
-----> n = 5, leftSide = 0.2459443806488943, difference = 0.
↪371089482071115
-----> n = 10, leftSide = 0.1252636387072129, difference =_
↪0.375923594920059
-----> n = 100, leftSide = 0.01012278230045116, difference_
↪= 0.241065860850507
-----> n = 200, leftSide = 0.004981838752162848, difference_
↪= 0.199046738584674
-----> n = 500, leftSide = NaN, difference = NaN

```

```
-----  
x0=2.356194490192345, y0=1.5707963267948966, Power=0.3, □  
  ↪ lamda=1, q=0.25  
-----
```

```
sin(x*y)
```

```
-----> n = 1, leftSide = 0.5271717580108257, difference = 0.  
  ↪ 472828241989174  
-----> n = 5, leftSide = 0.1977075625508048, difference = 0.  
  ↪ 419326300169205  
-----> n = 10, leftSide = 0.17545000849986037, difference = □  
  ↪ 0.325737225127412  
-----> n = 100, leftSide = 0.02264908125807985, difference □  
  ↪ = 0.228539561892878  
-----> n = 200, leftSide = NaN, difference = NaN  
-----> n = 500, leftSide = NaN, difference = NaN
```

```
cos(x+y)
```

```
-----> n = 1, leftSide = 0.666766673600017, difference = 0.  
  ↪ 333233326399983  
-----> n = 5, leftSide = 0.23998424116413997, difference = □  
  ↪ 0.377049621555870  
-----> n = 10, leftSide = 0.11145546263263517, difference = □  
  ↪ 0.389731770994637  
-----> n = 100, leftSide = 0.009950631514490937, difference □  
  ↪ = 0.241238011636467  
-----> n = 200, leftSide = NaN, difference = NaN  
-----> n = 500, leftSide = NaN, difference = NaN
```

```
-----  
x0=2.356194490192345, y0=1.5707963267948966, Power=0.3, □  
  ↪ lamda=1, q=0.5  
-----
```

```
sin(x*y)
```

```

-----> n = 1, leftSide = 0.5653606011950689, difference = 0.
↪434639398804931
-----> n = 5, leftSide = 0.07432229274307223, difference =
↪0.542711569976937
-----> n = 10, leftSide = 0.08060332438948326, difference =
↪0.420583909237789
-----> n = 100, leftSide = 0.0112451067475704, difference =
↪0.239943536403388
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.4778374955528847, difference = 0.
↪522162504447115
-----> n = 5, leftSide = 0.13044635524603976, difference =
↪0.486587507473970
-----> n = 10, leftSide = 0.05821192367045103, difference =
↪0.442975309956821
-----> n = 100, leftSide = 0.00499993931033893, difference
↪= 0.246188703840619
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

x0=2.356194490192345, y0=2.356194490192345, Power=0.3, lamda=0.
↪25, q=0.25

```

```

sin(x*y)
-----> n = 1, leftSide = 0.6925871453474989, difference = 0.
↪307412854652501
-----> n = 5, leftSide = 0.7584957733336617, difference =
↪-0.141461910613652
-----> n = 10, leftSide = 0.8789857151594607, difference =
↪-0.377798481532188

```

```

-----> n = 100, leftSide = 0.10758145989687384, difference
↪= 0.143607183254084
-----> n = 200, leftSide = 0.05132398144856276, difference
↪= 0.152704595888274
-----> n = 500, leftSide = 0.019889821206692027, difference
↪= 0.135102077548142
cos(x+y)
-----> n = 1, leftSide = 0.04302847225921516, difference =
↪0.956971527740785
-----> n = 5, leftSide = 0.21735958970458044, difference =
↪0.399674273015429
-----> n = 10, leftSide = 0.39345934112815295, difference =
↪0.107727892499119
-----> n = 100, leftSide = 0.055348639049624435, difference
↪= 0.195840004101334
-----> n = 200, leftSide = 0.02771298567058151, difference
↪= 0.176315591666255
-----> n = 500, leftSide = 0.011089529016998372, difference
↪= 0.143902369737835

```

```

-----
x0=2.356194490192345, y0=2.356194490192345, Power=0.3, lamda=0.
↪25, q=0.5
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.687036222663277, difference = 0.
↪312963777336723
-----> n = 5, leftSide = 0.5934355620160883, difference = 0.
↪023598300703921
-----> n = 10, leftSide = 0.5624694469096623, difference =
↪-0.061282213282390
-----> n = 100, leftSide = 0.05480942320370796, difference
↪= 0.196379219947250

```

```

-----> n = 200, leftSide = 0.025905610387507827, difference =
↪= 0.178122966949329
-----> n = 500, leftSide = 0.009982890207115513, difference =
↪= 0.145009008547718
cos(x+y)
-----> n = 1, leftSide = 0.050930004607481885, difference =
↪0.949069995392518
-----> n = 5, leftSide = 0.015100386386186132, difference =
↪0.601933476333823
-----> n = 10, leftSide = 0.1926149215227114, difference =
↪0.308572312104561
-----> n = 100, leftSide = 0.02768495990109943, difference =
↪= 0.223503683249859
-----> n = 200, leftSide = 0.013857824420666817, difference =
↪= 0.190170752916170
-----> n = 500, leftSide = 0.005544849757553584, difference =
↪= 0.149447048997280

```

```

-----
x0=2.356194490192345, y0=2.356194490192345, Power=0.3, lamda=0.
↪5, q=0.25
-----

```

```

sin(x*y)
-----> n = 1, leftSide = 0.7103775500517433, difference = 0.
↪289622449948257
-----> n = 5, leftSide = 0.8400996538212805, difference =
↪-0.223065791101271
-----> n = 10, leftSide = 0.6119861519617393, difference =
↪-0.110798918334467
-----> n = 100, leftSide = 0.051410506737241124, difference =
↪= 0.199778136413717
-----> n = 200, leftSide = 0.025020872041063735, difference =
↪= 0.179007705295773
-----> n = 500, leftSide = NaN, difference = NaN

```

cos(x+y)

```
-----> n = 1, leftSide = 0.01619033567029482, difference = 0.983809664329705
-----> n = 5, leftSide = 0.36436684098431893, difference = 0.252667021735691
-----> n = 10, leftSide = 0.26214283494469137, difference = 0.239044398682581
-----> n = 100, leftSide = 0.027712292875384497, difference = 0.223476350275574
-----> n = 200, leftSide = 0.013861243745072993, difference = 0.190167333591764
-----> n = 500, leftSide = NaN, difference = NaN
```

x0=2.356194490192345, y0=2.356194490192345, Power=0.3, lamda=0.5, q=0.5

sin(x*y)

```
-----> n = 1, leftSide = 0.7090579463642067, difference = 0.290942053635793
-----> n = 5, leftSide = 0.5425020384013203, difference = 0.074531824318689
-----> n = 10, leftSide = 0.3450579219536573, difference = 0.156129311673615
-----> n = 100, leftSide = 0.02599518205331708, difference = 0.225193461097641
-----> n = 200, leftSide = 0.012581620450654696, difference = 0.191446956886182
-----> n = 500, leftSide = NaN, difference = NaN
```

cos(x+y)

```
-----> n = 1, leftSide = 0.09418888355280962, difference = 0.905811116447190
-----> n = 5, leftSide = 0.1732451421806875, difference = 0.443788720539322
```



```

-----> n = 10, leftSide = 0.13219821581911256, difference = 0.368989017808160
-----> n = 100, leftSide = 0.013857478599215869, difference = 0.237331164551742
-----> n = 200, leftSide = 0.006930788347494963, difference = 0.197097788989342
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=2.356194490192345, y0=2.356194490192345, Power=0.3, lamda=1, q=0.25
-----

```

sin(x*y)

```

-----> n = 1, leftSide = 0.6698891250551241, difference = 0.330110874944876
-----> n = 5, leftSide = 0.6082720715946032, difference = 0.008761791125406
-----> n = 10, leftSide = 0.30354265011280357, difference = 0.197644583514469
-----> n = 100, leftSide = 0.025121339875409543, difference = 0.226067303275548
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

cos(x+y)

```

-----> n = 1, leftSide = 0.2101981690253152, difference = 0.789801830974685
-----> n = 5, leftSide = 0.25730768810921967, difference = 0.359726174610790
-----> n = 10, leftSide = 0.13666230437207122, difference = 0.364524929255201
-----> n = 100, leftSide = 0.01386690093644301, difference = 0.237321742214515
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

```

-----
x0=2.356194490192345, y0=2.356194490192345, Power=0.3, lamda=1,
  ↪ q=0.5
-----
sin(x*y)
-----> n = 1, leftSide = 0.6542084126179514, difference = 0.
  ↪ 345791587382049
-----> n = 5, leftSide = 0.3584336915320941, difference = 0.
  ↪ 258600171187916
-----> n = 10, leftSide = 0.16452563031225964, difference =
  ↪ 0.336661603315013
-----> n = 100, leftSide = 0.012670741987398593, difference
  ↪ = 0.238517901163559
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN
cos(x+y)
-----> n = 1, leftSide = 0.017085590495459647, difference =
  ↪ 0.982914409504540
-----> n = 5, leftSide = 0.12966081548961508, difference =
  ↪ 0.487373047230395
-----> n = 10, leftSide = 0.06842715102822401, difference =
  ↪ 0.432760082599048
-----> n = 100, leftSide = 0.0069292372673999654,
  ↪ difference = 0.244259405883558
-----> n = 200, leftSide = NaN, difference = NaN
-----> n = 500, leftSide = NaN, difference = NaN

```

7.3 Using SageMath

For completeness, we give below (without numbers) SageMath code that can be used for the Bivariate case approximation.

```

[ ]: var('x y') #declare x and y as variables
RR.scientific_notation(True)
powers = [3/10, 1/2, 7/10]
lamdas = [1/4, 1/2, 1] #deformation parameter lamda over (0,
↳1] - these are the beta values in the formula
qs = [1/4, 1/2] #deformation coefficient

funcs = [sin(x*y), cos((x+y)/2)] #choice of functions
a = -pi #the interval
b = pi #the interval
x0s= [pi/4, pi/2, 3*pi/4]
y0s= [pi/4, pi/2, 3*pi/4]

#####
for x0 in x0s:
#####
    for y0 in y0s:
↳
↳#####
        for power in powers: #going over various powers↳
↳for 1/n^power
↳
↳#####
            for lamda in lamdas: #going over each lamda value
↳
↳#####
                for q in qs: #going over each q value
↳
↳#####
                    #the activation function
                    phi(x) = (e^(lamda*x)-q*e^(-lamda*x))/
↳(e^(lamda*x)+q*e^(-lamda*x)) #formula 17.1
                    G(x) = 1/4*(phi(x+1) - phi(x-1)) ↳
↳#formula 17.9

```

```

#MultiG(x, y) = G(x)*G(y)
↪#formula 17.58
    print()
    print()
    ↪
↪print("-----
        print("(x0, y0) = (" + str(x0) + ", " +
↪str(y0)+ "), Power = "+ str(power)+ ", lamda = "+
↪str(lamda) + ", q = " + str(q))
    ↪
↪print("-----

    ↪
↪#####
    for i in range(len(funcs)):
    ↪
↪#####
        f(x,y)=funcs[i]
        show(f(x,y))
        for n in [1, 4, 16, 128]:
            #below it uses formula 17.69
            #numerator = sum( sum( f(k1/n,k2/
↪n)*(G(n*x-k1)*G(n*y-k2)) for k2 in [ceil(n*a),..
↪,floor(n*b)]) for k1 in [ceil(n*a),..,floor(n*b)])
            #denominator = (sum(G(n*x-k1) for
↪k1 in [ceil(n*a),..,floor(n*b)]))*(sum(G(n*y-k2) for k2 in
↪[ceil(n*a),..,floor(n*b)]))
            #L(x,y) = numerator/denominator
            numerator = sum( sum( f(k1/n,k2/
↪n)*(G(n*x0-k1)*G(n*y0-k2)) for k2 in [ceil(n*a),..
↪,floor(n*b)]) for k1 in [ceil(n*a),..,floor(n*b)])
            denominator = (sum(G(n*x0-k1) for
↪k1 in [ceil(n*a),..,floor(n*b)]))*(sum(G(n*y0-k2) for k2 in
↪[ceil(n*a),..,floor(n*b)]))

```

```

                                #L = numerator/denominator

                                leftSide = abs(numerator/
↪denominator - f(x0,y0)).n()
                                val3 = 1/(n^power).n()

                                print("                n = "+str(n), " ,  ",
↪ left side = "+str(leftSide),
                                "\n                                \t 1/
↪n^("+str(power)+") = "+str(val3), "\t difference = ",
↪"+str(val3-leftSide))

```

8 Conclusion

Here we give numerical applications for the approximation by the parametrized, deformed activation function neural networks. The parametrized activation function kills much fewer neurons than the original one. The asymmetry of the brain is best expressed by deformed activation functions. So in this monograph we present extensive numerical applications of neural networks approximation, as they are presented for the first time in the recent monograph by the first author, titled “Parametrized, Deformed and General Neural networks”, Springer, Heidelberg, New York, 2023. That is confirming with numbers the theoretical results of this monograph.

References

- [1] George A. Anastassiou (2023) *Parametrized, Deformed and General Neural Networks*, Springer, Heidelberg, New York.
- [2] George A. Anastassiou, Razvan A. Mezei (2015) *Numerical Analysis Using Sage*, Springer, Heidelberg, New York.
- [3] The Sage Developers (2023) *the Sage Mathematics Software System (Version 9.5)* , <https://www.sagemath.org>.
- [4] StatisticalHelp (2023) *Precision*, <https://www.statsdirect.com/help/basics/precision.htm>

